MOVING COSTS AND THE MICROECONOMICS OF INTRA-URBAN MOBILITY

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This paper considers the role of moving costs in long-term optimal moving strategies from a theoretical microeconomic point of view. Using a model of intertemporal utility maximization, we arrive at conditions for (i) the optimal number of moves, (ii) the optimal moving dates, and (iii) the optimal consumption of housing services and other goods in the periods between moves. Also, comparative static results are obtained.

1. Introduction

In order to adjust the consumption of housing services, the household will in general have to go through an expensive residential relocation process, involving many different kinds of costs such as searching costs, transaction and direct moving costs, and possible upheaval and psychic costs [see Maclennan (1977, 1979), and Linneman and Graves (1983)]. Hence, moving is not too often undertaken and the household is faced with the problem of choosing an optimal level of housing consumption under the constraint that this consumption level has to stay constant for some period. This implies that the consumption of housing services at certain instants of time within that period may be suboptimal compared to relative prices, income and family composition.

This problem has been studied by Muth (1974). In his model, moves were treated as a stochastic process, exogenous to the decision of how much housing to consume. However, although it seems very reasonable to assume that a move is not perfectly foreseen by the household, and therefore has a stochastic element, there also seems to be an interesting problem of analysing when it is optimal for the household to move, i.e., change the level of housing consumption.

Previous literature in the field of residential mobility [e.g., Goodman (1976), Quigley and Weinberg (1977), Weinberg (1979), Weinberg, Friedman and Mayo (1981), Alperovich (1983)], has mostly concentrated

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on identifying various factors that may lead to a move, such as changes in income and price of housing (including rent rebates), changes in the preference for housing (due to the influence of household structure and family life cycle), and changes in the preference for residential location (change of job, environmental changes, changes in commuting costs etc.). In contrast, this paper deals with the basic economics of how moving costs may act as a determinant of whether a move is taken or not. Indeed, a move may not come about at all even though there are considerable changes of the kinds mentioned above, because moving costs are too high. Furthermore, it is important to note that a move is not always initiated by abrupt changes of any kind, but may be the result of long-term consumption planning based on evaluations of permanent-income.

Hence the aim of the paper is to make a contribution to the micro-economics of that part of intra-urban mobility that is explained by long-term planning. More precisely, the objective is, within the framework of a model of intertemporal utility maximization, to determine (i) the optimal number of moves, (ii) the optimal moving dates, and (iii) the optimal (constant) consumption of housing services and other goods in the period between moves. Problems (ii) and (iii) are investigated in section 2 and the full problem of (i), (ii) and (iii) is dealt with in section 3.

2. Optimal consumption profiles and optimal moving dates

2.1. Assumptions

I shall make the following assumptions.

A.1. The individual seeks to maximize utility in the planning period $[0, T]$ where $T > 0$, is the planning horizon. There is no bequest motive.

A.2. The individual has an instantaneous utility function $U = U(q(t), x(t))$, where $q(t)$ and $x(t)$ are the flow of housing services and the expenditure on all other consumer goods at instant $t \in [0, T]$, respectively. $U$ is a continuous, twice differentiable, increasing and strictly concave function defined for $q \in [0, \infty)$ and $x \in [0, \infty)$ and with

\[ \frac{\partial U(q(t), 0)}{\partial x(t)} = \frac{\partial U(0, x(t))}{\partial q(t)} = \infty, \quad \forall \, q, x. \]

A.3. Both $q$ and $x$ are normal goods.

\[ ^{1} \text{ Also other factors may interfere with the household's moving decisions. For instance, imperfections in the housing market, like rent- and price control, may distort geographical mobility and affect the household's moving strategy [Weinberg, Friedman and Mayo (1981, p. 346)].} \]
A.4. There is no impatience in consumption, and thus the utility function for the whole planning period may be written

\[ V = \int_0^T U(q(t), x(t)) \, dt. \]

A.5. The individual has an income stream, \( y(t) \), defined on \([0, T]\).

A.6. The individual has access to a perfectly functioning capital market where the rate of interest, \( i \), is a positive constant.

A.7. The price of housing services, \( p \), the price of other consumption goods (put equal to one), and the cost per move, \( m \), are strictly positive constants throughout the planning period.

A.8. The number of moves, \( n \), is given, and the individual moves at the dates \((\theta_1, \ldots, \theta_n) (0, T) \) (to be determined) and consumes a constant amount of housing services, \( \bar{q}_j \), in the period between moves \((j = 1, \ldots, n + 1)\) where \( \theta_{n+1} = T \). Later we shall assume \( n \) to be a free variable.

2.2. The optimization problem and the optimality conditions

The individual is faced with the following constrained maximization problem:

\[
\max_{\{q, x(t), \theta\}} V = \int_0^{\theta_1} U(\bar{q}_1, x_1(t)) \, dt + \cdots + \int_0^{\theta_j} U(\bar{q}_j, x_j(t)) \, dt + \cdots
\]

\[
+ \int_0^T U(\bar{q}_T, x_T(t)) \, dt, \quad \text{s.t.} \quad (1)
\]

\[
\int_0^{\theta_1} (p\bar{q}_1 + x_1(t)) e^{-it} \, dt + \cdots + \int_0^{\theta_j} (p\bar{q}_j + x_j(t)) e^{-it} \, dt + \cdots
\]

\[
+ \int_0^T (p\bar{q}_T + x_T(t)) e^{-it} \, dt + m(e^{-i\theta_1} + \cdots + e^{-i\theta_j} + \cdots + e^{-i\theta_n})
\]

\[
\leq \int_0^T y(t) e^{-it} \, dt, \quad (2)
\]

with dual variable denoted \( \lambda \).

\[ \text{This assumption is due to pure convenience only. If the subjective discount rate were strictly positive, the optimal consumption paths would be decreasing, constant or increasing according as the discount rate were greater than, equal to or smaller than the force of interest. By assuming that the subjective discount rate is zero, I choose to consider increasing paths. However, the results are symmetrical for decreasing paths.} \]
The first-order conditions for an interior solution are [in addition to (2)]

\[
\int_{\theta_{j-1}}^{\theta_j} U_{\theta j}' \, dt = \lambda \int_{\theta_{j-1}}^{\theta_j} e^{-u} \, dt, \quad j=1,\ldots,n+1, \tag{3}
\]

where \( U_{\theta j}'(t) = (\partial U/\partial \theta j) \),

\[
U_{\theta j}(t) = \lambda e^{-u}, \quad j=1,\ldots,n+1, \tag{4}
\]

where \( U_{\theta j}'(t) = (\partial U/\partial x_j(t)) \),

\[
U(\bar{q}_j, x_j(\theta)) - U(\bar{q}_{j+1}, x_{j+1}(\theta)) = \lambda e^{-i\theta_j} \{ p(\bar{q}_j - \bar{q}_{j+1}) + x_j(\theta) - x_{j+1}(\theta) - \mu \}, \quad j=1,\ldots,n+1. \tag{5}
\]

Dividing (3) by \( \int_{\theta_{j-1}}^{\theta_j} U_{\theta j}' \, dt = (\theta_j - \theta_{j-1}) \) and eliminating \( \lambda \) from (4) we arrive at

\[
\frac{\int_{\theta_{j-1}}^{\theta_j} U_{\theta j}' \, dt/(\theta_j - \theta_{j-1})}{U_{\theta j}} = p \frac{\int_{\theta_{j-1}}^{\theta_j} e^{-u} \, dt/(\theta_j - \theta_{j-1})}{e^{-ik}}, \quad j=1,\ldots,n+1 \text{ and } k \in [0,T]. \tag{6}
\]

Proposition 1. In order to maximize the utility, consumption should be adjusted such that the ratio between the average marginal utility of housing consumption in a period \([\theta_{j-1}, \theta_j]\) between two moves and the marginal utility of other consumption at any other instant \(k \in [0,T]\), is equal to the ratio between the average price of housing services in the period \([\theta_{j-1}, \theta_j]\) and the price of all other goods at date \(k\).

Alternatively, this condition may be given the following form:

\[
\frac{\int_{\theta_{j-1}}^{\theta_j} U_{\theta j}' \, dt/(\theta_j - \theta_{j-1})}{\int_{\theta_{j-1}}^{\theta_j} U_{\theta j}' \, dt/(\theta_j - \theta_{j-1})} = p. \tag{7}
\]

Proposition 1a. In order to maximize the utility, consumption should be adjusted such that the ratio between the average marginal utility of housing consumption in a period \([\theta_{j-1}, \theta_j]\) between two moves and the average marginal utility of consumption of other goods in the same period, is equal to the ratio between the average period-prices of the two goods.

This proposition states that, even though it is not possible to equate the marginal rate of substitution (MRS) between housing and other goods to the
price ratio at every instant in the period, one should equate the average 'period-MRS' to the price-ratio.

Condition (5) simply states that in order to maximize utility, the moving dates should so be chosen, that the utility from making a marginal postponement of a move [the left-hand side of (5)] is equal to the utility-value of the cost of the marginal postponement [right-hand side of (5)].³

2.3. The optimal consumption programme

As to the optimal consumption paths, the following proposition can be made.

Proposition 2. Under the assumptions of the model, a consumption programme solving the utility maximization problem will have the following characteristics: (a) in the period between two moves, [\(\theta_{j-1}, \theta_j]\), the function \(q_j(t)\) is a constant and the function \(x_j(t)\) is increasing monotonically in \(t\); (b) for the whole planning period, \([0, T]\), the function \(x(t), t \in [0, T]\), will be (a) discontinuous with a downward jump at the moving dates if \(\partial^2 U/\partial x \partial q > 0\), (b) discontinuous with an upward jump if \(\partial^2 U/\partial x \partial q < 0\), (c) continuous for all \(t \in [0, T]\) if \(\partial^2 U/\partial x \partial q = 0\); (y) the average period-consumption of housing services and other goods are increasing successively from period to period.

Proofs. (a) Condition (4) implies that \((\partial U_x/c_{1t})/\partial t < 0\), \(t \in [\theta_{j-1}, \theta_j]\) since \(\tilde{q}_j\) is constant in this interval. From concavity, A.2, we have \((\partial x(t)/\partial t) > 0\), \(t \in [\theta_{j-1}, \theta_j]\).

(β) Condition (4) implies that \(U_x(t)\) be continuously decreasing in \([0, T]\). If \(\partial^2 U/\partial x \partial q < 0\), the upward jumps of \(q(t)\) at the moving dates lead to downward jumps of \(U_x(t)\). For \(U_{x(t)}\) to be continuous over these dates, this will have to be compensated by downward jumps of \(x(t)\) at these dates. If \(\partial^2 U/\partial x \partial q > 0\), compensation requires upward jumps of \(x(t)\). If \(\partial^2 U/\partial x \partial q = 0\), no jumps in \(U_{x(t)}\) occur and \(x(t)\) is continuously increasing in \([0, T]\).

(γ) From (3) and (4) and the strict convexity of the discount-function,

\[
\frac{\int^{\theta_{j+1}}_{\theta_{j-1}} U_q \, dt}{\theta_{j+1} - \theta_{j-1}} = \frac{\lambda p \int^{\theta_{j+1}}_{\theta_{j-1}} e^{-it} \, dt}{\theta_{j+1} - \theta_{j-1}} > \frac{\lambda p \int^{\theta_{j+1}}_{\theta_{j-1}} e^{-it} \, dt}{\theta_{j+1} - \theta_{j}} = \frac{\int^{\theta_{j+1}}_{\theta_{j-1}} U_q \, dt}{\theta_{j+1} - \theta_{j}},
\]

and

\[
\frac{\int^{\theta_{j}}_{\theta_{j-1}} U_x \, dt}{\theta_{j+1} - \theta_{j-1}} = \frac{\lambda \int^{\theta_{j}}_{\theta_{j-1}} e^{-it} \, dt}{\theta_{j+1} - \theta_{j}} > \frac{\lambda \int^{\theta_{j}}_{\theta_{j-1}} e^{-it} \, dt}{\theta_{j+1} - \theta_{j}} = \frac{\int^{\theta_{j}}_{\theta_{j-1}} U_x \, dt}{\theta_{j+1} - \theta_{j}},
\]

³The change in utility may be either positive or negative and so may the costs of postponement.
i.e., the average marginal utilities of $q$ and $x$ are decreasing from period to period such that (7) is fulfilled. But in view of A.2 and A.3, this implies that the average consumption of both $q$ and $x$ must increase over time.

2.4. Optimal moving dates

In order to determine the optimal moving dates, it is necessary to specify the utility function. I shall, therefore, assume that the utility function is time-additive and log-linear, so that we may write

$$V(\bar{q}, x(t), \theta) = \int_0^{\theta_1} \alpha \ln \bar{q}_t \, dt + \cdots + \int_0^{\theta_j} \alpha \ln \bar{q}_j \, dt + \cdots + \int_0^{T} \alpha \ln \bar{q} \, dt + \int_0^{T} (1-\alpha) \ln x(t) \, dt,$$

where $0 \leq \alpha \leq 1$.

Solving the maximization problem we find

$$\bar{q}_j^* = \frac{Y - M \theta_j - \theta_{j-1}}{T \theta_j - \theta_{j-1}} e^{-\alpha \theta_j} d\theta, \quad j = 1, \ldots, n+1, \tag{9}$$

$$x^*(t) = (1-\alpha) \frac{Y - M \theta_j}{T \theta_j} e^{\alpha \theta_j} \quad j = 1, \ldots, n+1, \tag{10}$$

where $Y = \int_0^T y(t) e^{-\alpha \theta_j} d\theta$ and $M = m \sum_{j=1}^n e^{-\alpha \theta_j}$.

Substituting $q_j^*$ and $x^*(t)$ into (8), taking the derivative with respect to $\theta_j$, and putting this equal to zero, we have

$$\frac{\partial V(q^*, x^*, \theta)}{\partial \theta_j} = \alpha \left\{ \frac{T \theta_j \theta_{j-1}}{Y - M} e^{-\alpha \theta_j} + \ln \frac{g_j}{e^{g_j} - 1} - \frac{g_j}{e^{g_j} - 1} - \ln \frac{h_j}{e^{h_j} - 1} + \frac{h_j}{e^{h_j} - 1} \right\} = 0, \tag{11}$$

where $g_j = i(\theta_j - \theta_{j-1})$ and $h_j = i(\theta_{j+1} - \theta_j)$.

However, in order to gain insight, assume that the moving costs are given by the function

$$m(t) - m(0) e^{rt}, \quad r \geq 0. \tag{12}$$
In this case the term, $T \cdot \pi m \cdot e^{-(i-r)\theta j}/(Y-M)$ in (11) is replaced by

$$\frac{T \cdot p(i-r) \cdot m(0) \cdot e^{-(i-r)\theta j}}{Y-M'}$$

(13)

where $M' = m(0) \sum_{j=1}^{n} e^{-(i-r)\theta j}$.

Then, we have the following proposition.

**Proposition 3.** Under the assumptions of the model, the individual maximizes utility by making (a) equidistant moves, provided $r = i$; (b) more and more frequent moves, provided $r < i$; (γ) less and less frequent moves provided $r > 1$.

**Proofs.** (a) If $r = i$, (13) vanishes from (11) and the following solutions emerge:

$$\theta_{j}^{\ast} = \frac{\theta_{j+1}^{\ast} - \theta_{j-1}^{\ast}}{2} + \theta_{j-1}^{\ast}, \quad j = 1, \ldots, n.$$  

(14)

Upon substituting successively we find

$$\theta_{j}^{\ast} = \frac{jT}{n + 1}, \quad j = 1, \ldots, n,$$

(15)

i.e., moves should be made equidistantly.

(β) If $r < i$, equidistant moving implies

$$\frac{(\partial V(q^{\ast}, x^{\ast}(t), (\theta_{j+1}^{\ast} + \theta_{j-1}^{\ast})/2))}{\partial \theta_{j}} = \frac{T \cdot p(i-r) \cdot m(0) \cdot e^{-(i-r)\theta j}}{Y-M'} > 0.$$  

(16)

Since $V(q^{\ast}, x^{\ast}(t), \theta)$ is strictly concave in $\theta$, $\theta_{j}^{\ast}$ lies to the right of the midpoint between $\theta_{j-1}^{\ast}$ and $\theta_{j+1}^{\ast}$,

$$\theta_{j}^{\ast} > \frac{\theta_{j+1}^{\ast} - \theta_{j-1}^{\ast}}{2} + \theta_{j-1}^{\ast}, \quad j = 1, \ldots, n.$$  

4The proof of this is long and tedious and may be obtained from the author upon request. The same goes for the proof of the strict concavity of (17) with respect to $n$. Also proofs for the signs of $A$ and $H$ in the proof of Proposition 4 and of $(\partial V/\partial m)_{m=0}$ in the proof of Proposition 5, are available from the author.
Rearranging terms we get
\[ \theta_j^* - \theta_{j-1}^* > \theta_{j+1}^* - \theta_j^*, \quad j = 1, \ldots, n, \]
i.e., moves should be made more and more frequently. This is also the solution if moving costs are constant (i.e., \( r = 0 \)).
(γ) If \( r > 1 \), it follows by parallel arguments to those of (β) that moves should be made less and less frequently. □

3. Optimal number of moves

The algorithm for finding the optimal number of moves is straightforward: for each number of moves, \( n \), find the optimal values of \( x \) and \( q \) and the optimal moving dates and calculate the utility \( V \). Choose that number, \( n^* \) that gives maximal utility i.e., such that
\[ V(q^*(n^*), x^*(n^*), \theta^*(n^*)) \geq V(q^*(n'), x^*(n'), \theta^*(n')), \quad n' = 1, 2, \ldots, \infty. \]

3.1. The case of the log-linear utility function

Consider the case of the log-linear utility function with \( m(t) = m(0)e^{it} \). Substituting \( q_j^* \), \( x_j^*(t) \) and \( \theta_j^* \) from (9), (10) and (15) into (8), we arrive at
\[ \alpha(T - mn) = T \ln \left( \frac{\alpha T (Y - mn)}{(n+1)(1 - e^{-z})} \right) + \frac{n}{2} z + T(1 - \alpha) \ln \left( \frac{(1 - \alpha)(Y - mn)}{Tp} \right) + \frac{i T}{2} \]
where \( z = iT/(n + 1) \).

The optimal number of moves, \( n^* \), is determined by
\[ \frac{\partial V}{\partial n} = \alpha T \left[ - \frac{1}{n + 1} + \frac{z}{n + 1} \left( \frac{1}{1 - e^{-z}} - \frac{1}{2} \right) \right] - \frac{(1 - \alpha)Tm}{Y - mn} = 0. \]

As to comparative results, we have the following proposition.

Proposition 4. Under the assumptions of the model, the individual maximizes utility by making more moves; the higher is his income; the higher is his preferences for housing services; the higher is the rate of interest; the larger is the planning period; and the lower are moving costs.
Proof. Take the total differential of $\partial V/\partial n$ and obtain

$$dn = \frac{1}{H} \left\{ \frac{TY}{(Y-mn)^2} dm - \frac{m}{Y-mn} dY - T \frac{\partial V}{\partial n} d\alpha - \frac{T}{(n-1)^2} A d\alpha + \frac{1}{(n+1)^2} A dT \right\},$$

where

$$H = \frac{\partial^2 V}{\partial n^2} = -\frac{1}{(n+1)} \left[ -1 + 2z \left( \frac{1}{1-e^{-\frac{\alpha}{n}}} - \frac{1}{2} \right) - z^2 \frac{e^{-\frac{\alpha}{n}}}{(1-e^{-\frac{\alpha}{n}})^2} \right] < 0,$$

$$A = \left[ \frac{1}{1-e^{-\frac{\alpha}{n}}} - \frac{1}{2} - \frac{ze^{-\frac{\alpha}{n}}}{(1-e^{-\frac{\alpha}{n}})^2} \right] > 0, \quad z = \frac{iT}{n+1}.$$ 

Thus we have

$$\frac{\partial n}{\partial m} < 0, \quad \frac{\partial n}{\partial Y} > 0, \quad \frac{\partial n}{\partial \alpha} > 0, \quad \frac{\partial n}{\partial \alpha} > 0, \quad \frac{\partial n}{\partial T} > 0. \quad \square$$

Furthermore we have the following proposition.

Proposition 5. Under the assumptions of the model, the optimal number of moves approaches infinity as moving costs approach zero.

Proof. The proposition follows from the fact that $(\partial V/\partial n)_{m=0} > 0$ for $iT > 0$, i.e., since the derivative of $V$ with respect to $n$ is positive for all $n$, the utility will be increasing for each extra move irrespective how high $n$ is, provided moving costs are zero. \quad \square

3.2. An example

To illustrate the model, consider a household with a life-time income of $500,000 planning for a 60-year period and assume that the rate of interest is equal to 5%, that $x=0.15$ and that moving costs are equal to $1000$. In this case the optimal number of moves is equal to 3. Since moving costs are assumed constant throughout the planning period, moves will be made more and more frequently. However, in the chosen example, equidistant moving turns out to be a very good approximation of the optimal moving dates. Hence the household will move after 15 years, after 30 years and after 45 years.

In tables 1 and 2 the optimal number of moves are calculated under various assumptions about moving costs, life-time income and the rate of interest.
Table 1
Optimal number of moves when $i=0.05$, $a=0.15$, $T=66$.

<table>
<thead>
<tr>
<th>Life-time income</th>
<th>500</th>
<th>1000</th>
<th>2000</th>
<th>4000</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.000</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>500.000</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>900.000</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2
Optimal number of moves when $i=0.1$, $a=0.5$, $T=60$.

<table>
<thead>
<tr>
<th>Life-time income</th>
<th>500</th>
<th>1000</th>
<th>2000</th>
<th>4000</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.000</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>500.000</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>900.000</td>
<td>9</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

For all these cases equidistant moving is a good approximation except for the case of $m=\$500$ and $Y=\$100,000$. In this case the optimal moving dates are $\theta_1^* = 21$ and $\theta_2^* = 42$.

4. Conclusions

The main conclusions of this paper are summarized in Propositions 1 through 5. Existing literature gives some indications on some of these conclusions. For instance, Weinberg, Friedman and Mayo (1981, p. 346) report that the costs of moving appear to be highly significant in influencing rates of residential search and mobility, and indicate that the higher mobility in Phoenix than in Pittsburgh is attributable to this factor. Furthermore, the tendency for renters to move more frequently than home-owners has been attributed to the fact that moving costs are much lower for renters than for homeowning households. A family already owning a home is less likely to move because it has made a major capital investment, costly to turn over [Weinberg (1979, p. 222), Maisel (1968, p. 99)]. Also, the lower frequency of moves among older households [see e.g., Kaluzny (1975, p. 270), Alperovich (1983, p. 288)] may be attributed to possible increasing psychic costs of moving as one gets older and grows more attached to a neighbourhood [Goodman (1976, p. 861)]. It may further be attributed to the shorter time
period over which to realize any adjustment benefits [Linneman and Greaves (1983, p. 274)].

However, existing literature is more inconclusive on the positive relationship between the level of income and the number of moves. For instance, Abu-Lughod and Foley (1966, p. 183) found that movers have lower income than non-movers and Weinberg (1979, p. 241) found that the middle income group appeared most mobile. However, in the first study, the effects of different tenure had not been taken into account and in the second study Weinberg himself does not put too much emphasis on his result because his measure of income is not particularly good (p. 341). A more firm result is obtained in Weinberg, Friedman and Mayo (1981, p. 345). Here, it is concluded that both the benefits of moving and the frequency of moving are less for low-income households as compared to households with higher incomes.

References


