

# Playing monopoly with the government: Time inconsistency of industrial policy revisited

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## Abstract

Tornell (1991) analyzes how strategic investment behavior by the protected firm renders the optimal employment-enhancing industrial policy time inconsistent when the government's demand for employment is perfectly inelastic and the product demand is perfectly elastic. By allowing more general functions in describing the government's demand for employment, we show that time inconsistency prevails, but that its cause changes from strategic behavior by the firm to governmental opportunism. Moreover, we show that the form of the time inconsistency problem depend on demand conditions and production technology, and that increasing the number of plants of the regulated monopoly may increase the time inconsistency problem. (102 words)

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## 1 Introduction

Governments may want to support domestic firms by many reasons. The support can be aimed at enhancing economic efficiency, to facilitate a more preferred income distribution, or both. Some programs are intended to be temporary (e.g., protection of "sunrise" and

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"sunset" industries) while others are meant to be more permanent. While some programs are aimed at providing domestic production of goods that would otherwise not have been produced in the country<sup>1</sup> (or, less extremely formulated, the production would have been too low without the support), it should be fair to say that most support programs are aimed at providing employment in "problem regions" and "problem industries."<sup>2</sup>

Suppose that the government proposes, say, a wage subsidy in order to increase the labor demand of some firm. Moreover, suppose that the firm is about to sink some investment in production capital, and that the firm's choice of investment level will affect the profitability of hiring workers at the subsequent production stage. First consider the effects of a proposed wage subsidy assuming that the government can commit not to change the policy after the firm has chosen the investment level. Clearly, the wage subsidy will make the firm demand more labor for any level of capital. Moreover, the subsidy will make labor cheaper relatively to capital, and the firm will adjust its capital stock accordingly, in a way that increases the labor demand even more.<sup>3</sup>

If the government is unable to commit not to change the proposed subsidy after the firm has made the investment, then what has been known as a *time inconsistency problem* arises. Time inconsistency of protectionist programs stems from the sequential nature of 'protectionist games' and have been analyzed by many scholars (see e.g. Tornell, 1991; Torsvik, 1993; Staiger and Tabellini, 1987; and Matsuyama, 1990). Using the language of game theory, time inconsistency refers to the failure of solution of the commitment game to be an equilibrium of the non-commitment game (see e.g. Fudenberg and Tirole, 1991, pp.74-75).

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<sup>1</sup>This line of reasoning has perhaps most notably been used as an argument for supporting military defense industries. However, the argument has been used in other industries as well (e.g. in agriculture).

<sup>2</sup>The programs also differ according to what kind of problems the firm or industry in question faces. Sometimes the support is tailored to rescue a firm or an industry from bankruptcy, while in other cases the problem is that it is not profitable for a firm or an industry to maintain as high a production or employment as the government prefers. Furthermore, different problems combine with different economical and political environments to call for different policy implementations. The government might for instance be restricted not to use certain policy instruments (e.g., labor subsidies, tariffs or investment subsidies), or the particular design of certain instruments can be restricted (e.g., production subsidies may have to be proportional to the produced quantity), by legal or other reasons.

<sup>3</sup>It is unclear, however, whether the subsidy will increase the investments or not. The substitution effect suggests that the investments will decrease, and an output effect (due to lower costs) suggesting that the investments will increase. The total effect depend, among other things, on the form of the product demand.

Time inconsistency may take different forms. The government may announce a protection plan that, if the protected firm does what is supposed, is self-enforcing (sequentially rational, time consistent). However, if the government is unable to commit to the plan, the protected firm may induce alterations by taking actions that they would not find profitable if the government could commit. In our example, the firm may induce increased subsidies at the production stage by decreasing (or, sometimes, by increasing) its investments. This form of the time inconsistency problem has been analyzed by Tornell (1991) and can be dubbed *strategic investments by the firm*.

In addition, lack of commitment can tempt the government to change the proposed policy after investments are sunk, even if the firm does what it is supposed to do. Since a subsidy has a greater effect when it can affect the firm's choice of investment, then, typically, the optimal wage subsidy is higher before the firm invests than afterwards, *ceteris paribus*. This form of time inconsistency can be dubbed *governmental opportunism* and has been studied by Torsvik (1993).<sup>4</sup>

One of the objectives of this paper is to establish a link between these two sources of time inconsistency, and we will see that both can be reproduced in a single framework. Intuitively, the form of time inconsistency analyzed by Tornell (1991) arises because the government cannot commit not to increase the subsidy, while the problem studied by Torsvik (1993) is caused by the inability to commit not to *decrease* the subsidy (both changes relative to the subsidy level that arises when the government can commit).

Although the problem of time inconsistency seems to be significant, its consequences have not been much explored.<sup>5</sup> Tornell (1991), whose analysis has the most detailed description of production and markets, assumes unlimited product demand at the given price, and that the government is going to maintain a given number of jobs, at any costs. Another purpose of this paper is to investigate how restrictive these assumptions are. In particular, we will show that these two assumptions combine to exaggerate the underinvestment problem resulting from time inconsistency. The analysis will also extend to two-plant firms where only employment in one of them is of value to the government.

The paper proceeds as follows: First, in section 2 we set up the basic model and

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<sup>4</sup>Torsvik proposes time inconsistency as an explanation why governments often use investment subsidies when the declared aim is to induce employment.

<sup>5</sup>In addition to the studies already mentioned, see Corden (1974), Zysman and Tyson (1983), Kydland and Prescott (1977) and Schelling (1960) for studies of variants of the time inconsistency problem.

discuss in relatively general terms the nature of the basic time inconsistency problem. We also characterize the equilibria of the games corresponding to the cases of policy precommitment and non-commitment, respectively. Next, in section 3, we study a special case of perfectly elastic product demand, and show how time inconsistency vary with the structure of the government's demand for employment. In section 4 we analyze another special case, that of perfectly inelastic demand for employment, and reach some conclusions regarding how time inconsistency vary with product demand and production technology. In section 5 we extend the basic model to analyze a regulated two-plant monopoly, and show that the time inconsistency problem may aggravate in that case. In section 6 we study optimal employment policy in a region consisting of firms in industries not competing directly with each other, and derive some principles of optimal discriminatory policy. Section 7 concludes the paper.

## 2 The model

Consider a firm who has a production function given by  $x = x(K, N)$ , where  $x$  is output,  $K$  is the amount of capital employed and  $N$  is the number of workers employed. Product demand is represented by the inverse demand function  $p = p(x)$ . Moreover, the firm faces a market price of capital equal to  $r$ , and a wage rate  $w$ , yielding the following expression for profit:

$$\pi(N, K) = p(x)x - rK - wN \quad (1)$$

There is also a government who has preference for employment in the firm, and who may subsidize use of labor to increase the employment. The government's objective is to maximize welfare given by

$$W(N) = V(N) - (w_0 - w)N, \quad (2)$$

where  $w_0$  is the going wage rate. The term  $V()$  is the government's utility from employment in the firm, while the second term is the cost of subsidizing employment in the firm. The subsidy per worker equals  $w_0 - w$ .

To save space we will simply assume that the functions  $W()$  and  $p()$  are 'well-behaved' in the sense that they give rise to interior solutions. As a consequence, no time will be spent on second order conditions.

We will consider two games, who differs only in the timing of the decisions. The first game, subsequently referred to as the *commitment game*, goes as follows:

1. The government decides the price of labor  $w$
2. The firm decides its investment  $K$
3. The firm hires labor  $N$  and production takes place

Solving the commitment game yields the following proposition:

**Proposition 1** *When the government can commit to a policy before the firm makes its investment, the equilibrium is characterized by:*

$$[p(x) - xp'(x)] x_N - w = 0, \quad (3)$$

$$[p(x) - xp'(x)] x_K - r = 0 \text{ and} \quad (4)$$

$$[V'(N) - (w_0 - w)] N'_C(w) + N = 0, \text{ where} \quad (5)$$

$$N'_C(w) = \frac{R'(x)x_{KK} + R''(x)(x_K)^2}{R'(x)^2 [x_{NN}x_{KK} - (x_{NK})^2] + R'(x)R''(x) [x_{NN}(x_K)^2 + x_{KK}(x_N)^2 - 2x_{NK}x_Nx_K]}$$

and  $R(x) \equiv xp(x)$ .

**Proof.** As usual we start by solving the last stage. Given the wage rate  $w$  and investments  $K$ , the firm hires labor to maximize profit given by (1). The corresponding first-order condition is equation (3) in the proposition. Next, in step 2 the firm designs investments to maximize profits, yielding a similar first-order condition (4). Equation (3) and (4) now implicitly define  $N$  and  $K$  as functions of  $w$ , denoted  $N_C(w)$  and  $K_C(w)$ . Knowing these functions, the government's problem is to maximize welfare given by equation (2), subject to the constraint that  $N = N_C(w)$ . The first-order condition is equation (5) above, and the expression (6) for the derivative of  $N_C(w)$  is obtained by differentiating through the two equations (3) and (4), then reorganizing. ■

Next, consider a game defined by the following sequence of moves:

1. The firm decides its investment  $K$
2. The government decides the price of labor  $w$
3. The firm hires labor  $N$  and production takes place

This game will subsequently be referred to as the *non-commitment game*. The characteristic difference between the two games is the sequence of determination of investment  $K$  and price of labor  $w$ . In the commitment game the government set (i.e., commit to) the wage  $w$  before investment is made. In the non-commitment game, the government do not set (or commit to) the policy until after the firm has made the investment.

The equilibrium of the non-commitment game is characterized in the following proposition:

**Proposition 2** *If the policy is determined after investment is made, the equilibrium is characterized as follows:*

$$[p(x) - xp'(x)] x_N - w = 0,$$

$$[V'(N) - (w_0 - w)] N'_N(w; K) + N = 0, \quad \text{and} \quad (6)$$

$$[p(x) - xp'(x)] x_K - r - \frac{dw}{dK} N = 0, \quad (7)$$

where

$$N'_N(w; K) = \frac{1}{R'(x)x_{NN} + R''(x)(x_N)^2}$$

and  $\frac{dw}{dK}$  is obtained by differentiating (3) and (6) and solving for  $\frac{dw}{dK}$ .

**Proof.** The labor demand function is the same as for the commitment case, implicitly defined by equation (3). This equation also defines  $N$  as an implicit function of  $w$  and  $K$ ;  $N_N(w; K)$ . Given investments  $K$ , the first-order conditions for optimal  $w$  is given by equation (6) above. This equation implicitly defines  $w$  as a function of  $K$ ; denoted  $w(K)$ . Given this function, equation (7) above is just the first-order conditions for optimal investment. The expression for  $N'_N(w; K)$  is obtained by partially differentiating through equation (3), then rearranging. ■

By comparing Propositions 1 and 2 we can identify the two possible sources of time inconsistency. First, equation (4) and (7) differ by the term  $-\frac{dw}{dK} N$ , which is an expression for the firm's ability to affect the policy by its choice of investment level. This term is the origin of what we in the introduction called *strategic behavior by the firm*. Second, the term  $N'_C(w)$  in equation (5) is replaced by  $N'_N(w; K)$  in equation (6). These terms can differ by two reasons: First, even if they are equal for one particular level of  $K$ , they will most likely differ for other values of  $K$ . This phenomenon can be referred to as *investment-induced change of policy*. Second, if we let  $K_C$  denote the equilibrium investment level of

the commitment game,  $N'_C(w)$  will typically differ from  $N'_N(w; K_C)$ , because the marginal effect of subsidies on labor demand depends on the firm's ability to substitute between labor and capital. This ability is clearly reduced when investment is sunk. This effect is what we in the introduction called *governmental opportunism*.

In the next two sections we will study the equilibria of these two games under different assumptions, while section 5 and 6 consider extensions to the basic model.

### 3 Strategic behaviour by the firm or governmental opportunism?

In this section we will study which assumptions make strategic behavior by the firm the major cause of time inconsistency, and which assumptions make governmental opportunism be the main source. To narrow the focus, we will make two simplifying assumptions. First, we will restrict attention to situations in which the product demand is perfectly elastic, that is,  $p(x)$  is normalized to unity (without loss of generality). Second, we will assume that the production function is of Cobb-Douglas type, that is,

$$x = K^\alpha N^\beta, \quad (8)$$

where  $\alpha + \beta < 1$ .

These assumptions facilitate computation of closed-form solutions of our two games. First we look at the commitment game:

**Proposition 3** *When demand is given by  $p \equiv 1$  and the production function is given by  $x = K^\alpha N^\beta$ , the equilibrium of the commitment game is characterized by:*

$$w = \frac{1 - \alpha}{\beta} [w_0 - V'(N)], \quad (9)$$

$$N = \left[ \frac{\beta^{1-\alpha} \alpha^\alpha}{r^\alpha w^{1-\alpha}} \right]^{\frac{1}{1-\alpha-\beta}} \quad \text{and} \quad (10)$$

$$K = \left[ \frac{\alpha^{1-\beta} \beta^\beta}{w^\beta r^{1-\beta}} \right]^{\frac{1}{1-\alpha-\beta}}. \quad (11)$$

**Proof.** Using (8) and the assumption that  $p \equiv 1$ , equations (3) and (4) can be written:

$$\beta \frac{x}{N} = w$$

and

$$\alpha \frac{x}{K} = r$$

Solving these two equations for  $N$  yields equation (10), while solving the same two equations for  $K$  yields (11). Next, differentiation of (10) yields

$$N'_C(w) = -\frac{1-\alpha}{1-\alpha-\beta} \frac{N}{w}.$$

Using this, equation (5) in Proposition 1 can be rewritten

$$- [V'(N) - (w_0 - w)] \frac{1-\alpha}{1-\alpha-\beta} \frac{N}{w} + N = 0.$$

Solving this equation for  $w$  yields equation (9). ■

A similar characterization is obtained for the non-commitment game:

**Proposition 4** *When demand is given by  $p \equiv 1$  and the production function is given by  $x = K^\alpha N^\beta$ , the equilibrium in the non-commitment case is characterized by:*

$$w = \frac{1}{\beta} [w_0 - V'(N)], \quad (12)$$

$$N = N_C(w; K) = \left[ \frac{\beta K^\alpha}{w} \right]^{\frac{1}{1-\beta}} \quad \text{and} \quad (13)$$

$$\alpha K^{\alpha-1} N^\beta - r = \frac{V''(N) \alpha \beta N^2 K^{\alpha-1}}{\beta^2 (1-\beta) K^\alpha - V''(N) N^{2-\beta}}. \quad (14)$$

**Proof.** Using (8) and the assumption that  $p \equiv 1$ , equation (3) yields equation (13). Differentiation of (13) yields

$$N'_N(w; K) = -\frac{N}{(1-\beta)\beta K^\alpha N^{\beta-1}} = -\frac{1}{1-\beta} \frac{N}{w}.$$

Using this expression, equation (6) in Proposition 2 can be written

$$- [V'(N) - (w_0 - w)] \frac{1}{1-\beta} \frac{N}{w} + N = 0.$$

Solving for  $w$  yields equation (12). Finally, by differentiating (12) and (13) and solving for  $dw/dK$ , we get the following expression:

$$\frac{dw}{dK} = -\frac{\alpha \beta N^{\beta-1} K^{\beta-1} V''(N)}{\beta^2 (1-\beta) K^\alpha N^{\beta-2} - V''(N)}.$$

Now equation (14) is easily obtained by inserting this expression into equation (7), using (8) and rearranging. ■

### 3.1 Two polar cases

In this subsection we will consider two specific cases. First, we will assume that  $V(N) \equiv vN$ , that is, the demand for employment is perfectly elastic. Since  $V'(N) = v$ , Propositions 3 and 4 immediately implies the following two corollaries:

**Corollary 1** *When  $V(N) \equiv vN$ , the commitment equilibrium wage is given by*

$$w = w_C \equiv \frac{(1 - \alpha)(w_0 - v)}{\beta}. \quad (15)$$

**Corollary 2** *When  $V(N) \equiv vN$ , the non-commitment equilibrium wage is given by*

$$w = w_N \equiv \frac{w_0 - v}{\beta}. \quad (16)$$

By comparing these results we see that the equilibrium wage is higher in the non-commitment case. A higher wage implies in turn that equilibrium investments and employment are both lower in the non-commitment equilibrium. Hence we have that non-commitment leads to underinvestment. Moreover, since underinvestment increases the cost of employment and the demand for employment is elastic, we also get underemployment.

In contrast to the case analysed by Tornell (1991), underinvestment is not caused by the firm's ability to alter the policy. In fact, in the setup of this subsection the firm cannot alter the policy ( $w$  is independent of  $K$ ). Underinvestment is caused by the firm's (correct) anticipation that the government wishes to alter its policy. This wish stems from the fact that the marginal cost of supplying employment when investment is sunk and when it is not differ.

The other extreme is to assume that employment demand is perfectly inelastic (cf., e.g., Tornell (1991)), that is,

$$V(N) = \begin{cases} -\infty & \text{if } N < \underline{N} \\ \underline{V} & \text{if } N \geq \underline{N} \end{cases}. \quad (17)$$

Then the government's problem can be written as follows:

$$\min_w \{(w_0 - w)N\} \quad s.t. \quad N(w) \geq \underline{N} \quad (18)$$

Propositions 5 and 6 characterize the equilibria of the commitment game and the non-commitment game, respectively, for this specific form of the function  $V(\cdot)$ .

**Proposition 5** *When employment demand is perfectly inelastic and the government can commit to a policy before investment is made, the equilibrium is characterized by:*

$$w = \left[ \frac{\beta^{1-\alpha} \alpha^\alpha}{r^\alpha \underline{N}^{1-\alpha-\beta}} \right]^{\frac{1}{1-\alpha}}, \quad (19)$$

$$N = \underline{N} \quad \text{and} \quad (20)$$

$$K = \left[ \frac{\alpha^{1-\beta} \beta^\beta}{w^\beta r^{1-\beta}} \right]^{\frac{1}{1-\alpha-\beta}}. \quad (21)$$

**Proof.** Equation (20) stems from the government's implicit employment demand function (17), equation (21) equals equation (11) and equation (19) is obtained by substituting  $\underline{N}$  for  $N$  in equation (10) and solving for  $w$ . ■

**Proposition 6** *When employment demand is perfectly inelastic and the government cannot commit to a policy before investment is made, the equilibrium is characterized by:*

$$K = \left[ \frac{\alpha(1+\beta)\underline{N}^\beta}{r} \right]^{\frac{1}{1-\alpha}}, \quad (22)$$

$$N = \underline{N} \quad \text{and} \quad (23)$$

$$w = \beta K^\alpha \underline{N}^{\beta-1}. \quad (24)$$

**Proof.** As in Proposition 5, (23) stems directly from (17). (24) is obtained by substituting  $\underline{N}$  for  $N$  in equation (3) and solving for  $w$ . (22) is obtained by substituting the right hand side of (24) into the profit function, which then becomes a function of  $K$  alone, and maximizing. ■

Also in this case time inconsistency leads to underinvestment. However, this underinvestment stems from strategically behavior by the firm; the firm underinvests to increase labor subsidies at the production stage. In contrast, the government is locked to the predetermined employment level, and can therefore not act opportunistically.

Before we leave the subject, we will briefly discuss the cases inbetween these polar cases. In doing so we will also allow for general production functions.

### 3.2 The case of general production and employment demand functions

More generally, by increasing the employment demand elasticity, we should expect underinvestment to prevail. In addition, underemployment will become an increasing problem. However, the cause of the problems will shift from strategically behaviour by the firm

to opportunistically behaviour by the government. This makes the analysis more complicated, since now the government's monopsony position in the demand for employment comes into action. Before we start, we will recapitulate some intuition from a similar problem; that of setting the optimal monopoly price when demand changes. As we know from the theory of industrial organization (see f.ex. Tirole (1988)), even with constant marginal costs, the monopoly price changes with demand conditions. From the similar theory of monopsony we know that even with constant marginal valuation of the good, optimal demand price changes with supply conditions. What investment does in this model is exactly to change the costs of "supplying employment" to the goverment.

If, as before,  $\pi = x(N, K) - rK - wN$  and  $W = V(N) - (w_0 - w)N$ , a perfectly elastic product demand allows us to write the implicit labor demand function as follows:

$$x_N - w = 0,$$

and the optimal price of labor is given by<sup>6</sup>

$$w(K) = w_0 - V'(N) - Nx_{NN} \quad (25)$$

Implicit differentiation yields the following expression for  $w'(K)$ :

$$w'(K) = -\frac{[Nx_{NNK} - x_{NK}]x_{NN} - Nx_{NNN}x_{NK} - V''(N)x_{NK}}{2x_{NN} + Nx_{NNN} + V''(N)}, \quad (26)$$

and we notice that it is far more difficult to find out the sign of the derivative in this case. To gain further insights we will once more subsitute a Cobb-Douglas production function for the more general production function. For the production function  $x = N^\alpha K^\beta$ , equation (26) can be simplified (after lots of messy, but straightforward calculus) to the following:

$$w'(K) = -\frac{x_{NK}V''(N)}{\alpha x_{NN} + V''(N)}. \quad (27)$$

As in the case of fixed demand for employment, a necessary conditions for time inconsistency to cause strategic underinvestment is that  $x_{NK} > 0$ .

Note that  $V'(N)$  can be interpreted as the inverse demand function for employment. By differentiating (27) with respect to  $V''(N)$  we find that  $\partial(w'(K))/\partial(V''(N)) < 0$ , which implies that the steeper is the demand for employment, the larger is  $w'(K)$ , and

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<sup>6</sup>To find the optimal labor subsidy, we maximize  $W$  subject to the implicit labor demand function  $px_N - w = 0$ . Letting  $\lambda$  denote the multiplier of the constraint, the Lagrangian can be written  $L(w, N, \lambda) = V(N) - (w_0 - w)N - \lambda[px_N - w]$ . Straightforward maximiation yields (after rearranging) equation (25).

the greater are therefore the firm's strategic incentives to underinvestment. The intuition behind this result is that the steeper is the demand curve, the more will the government be willing to adjust its policy to accomodate inadequate investment. In the limiting case where  $V''(N) = 0$ , the government have a perfectly elastic demand for employment, and (27) reduces to  $w'(K) = 0$ . However, this last result depends on the specific production function employed. If we utilize a skew Cobb-Douglas-function, for instance,

$$x(N, K) = (N - N_0)^\alpha K^\beta, \quad (28)$$

(26) reduces to

$$w'(K) = -\frac{\alpha\beta(N - N_0)^{\alpha-1}K^{\beta-1}N_0}{\alpha N - 2N_0} \begin{cases} > 0 & \text{if } 0 < N_0 < \frac{1}{2}\alpha N \\ = 0 & \text{if } N_0 = 0 \\ < 0 & \text{if } N_0 < 0 \end{cases} \quad (29)$$

when  $V''(N) = 0$ . Consequently, even with perfectly elastic demand for employment, time inconsistency can create underinvestment incentives.<sup>7</sup>

## 4 The effect of product demand and production technology

To narrow the focus on the effect of product demand and production technology, we will in this section assume that the government's demand for employment is fixed and equal to  $\underline{N}$ . As a consequence, the government will always select the wage  $w(K)$  that induces the firm to demand exactly  $\underline{N}$  units of labor. By reorganizing equation (3), the optimal wage can be written:

$$w(K) = [p(x(\underline{N}, K)) + x(\underline{N}, K)p'(x(\underline{N}, K))] x_N(\underline{N}, K). \quad (30)$$

Differentiating this expression yields:

**Proposition 7** *If the government have totally inelastic demand for employment, time inconsistency yields underinvestment iff*

$$w'(K) = [p + xp'] x_{NK} + [2p' + xp''] x_N x_K > 0. \quad (31)$$

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<sup>7</sup>Negative  $N_0$  can be interpreted as a technology where there is an initial endowment of labor. Consequently, for low production levels, the firm will not hire additional workers, but increase production by investing to increase the productivity of the existing labor force.

If the firm is a price taker,  $p' = p'' = 0$  and (31) reduces to  $w'(K) = px_{NK}$ , implying the following result:

**Corollary 3** *If product demand is perfectly elastic, time inconsistency lead to underinvestment iff the production factors are complementary.*

This result is similar to Tornell's (1991) main result. Tornell studied price subsidies instead of wage subsidies. However, for a given level of investment, the two instruments have the same qualitative effects.

If, in contrast, product demand is linear, then Proposition 7 implies the following result:

**Corollary 4** *If product demand is linear ( $p = a - bx$ ), time inconsistency lead to underinvestment iff*

$$w'(K) = -2bx_Nx_K + x_{NK}(a - 2bx) < 0 \quad (32)$$

Note that complementarity in the production function is no longer sufficient to create underinvestment incentives. To gain some further insights into the connections between demand structure and investment incentives, let us take a closer look at the case of Cobb-Douglas production function:  $x = K^\alpha N^\beta$ . It is straightforward to check that in this case equation (32) can be rewritten

$$w'(K) = -\frac{\alpha\beta x(4bx - a)}{NK} \begin{cases} > 0 & \text{if } x < \frac{a}{4b} \\ = 0 & \text{if } x = \frac{a}{4b} \\ < 0 & \text{if } x > \frac{a}{4b} \end{cases}, \quad (33)$$

clearly illustrating that the investment incentives are sensitive to the quantity of production and the demand conditions as well as the factor substitution conditions already known.

To sum up this section, we have found that the underinvestment incentives resulting from time inconsistency are stronger i) the less downward-sloping is the product demand; ii) the more complementary are the factors of production; and iii) the smaller is the production compared to the market size.

## 5 Many (atomistic) firms, specific instruments

In this section we will consider a variant of the basic model, by allowing the government to enhance employment by supporting different firms. This will have several implications.

First, each firm will have less power to change the policy, since the government now has outside options.<sup>8</sup> Second, due to the government's monopsony position in the market for employment, the optimal policy might involve subsidizing different firms at different rates.

The formal argument goes like this. Suppose that support can be granted to any of  $n$  firms that use the same labor force, which supplies workers at the wage rate  $w_0$ . For sake of simplicity we will further assume that the firms do not compete with each other on the product market. This can be accomplished by assuming that the market demand for their product is perfectly elastic, or that they sell their products in different markets (i.e., they produce different goods). Either way, each firm  $i$ 's problem can be stated as maximization of profit given by

$$\pi_i = p_i x_i(N_i, K_i) - rK_i - w_i N_i. \quad (34)$$

Moreover, welfare is (as before) defined by

$$W = V \left( \sum_i N_i \right) - \sum_i (w_0 - w_i) N_i. \quad (35)$$

Also in this case we distinguish between the commitment game and the non-commitment game. By the same kind of analysis as before, we find that the equilibrium of the commitment game must feature

$$w_i = w_0 - V' \left( \sum_i N_i \right) - p_i N_i \frac{\partial^2 x_i}{\partial N_i^2}. \quad (36)$$

This implies that two firms  $i$  and  $j$  should face the same wage if and only if

$$p_i N_i \frac{\partial^2 x_i}{\partial N_i^2} = p_j N_j \frac{\partial^2 x_j}{\partial N_j^2} \quad (37)$$

It is now easy to see that if the two firms have different technologies (e.g., Cobb-Douglas production functions with different parameters), they should normally be discriminated between. Note that this analysis can be extended to firms that compete with each other on the product market.

## 6 Two-plant monopoly with different employment demand

In some cases the protected firm has several production plants producing the same product (or a close substitute), and where the government put different value on employment in

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<sup>8</sup>Put alternatively, the government's monopsony power is now not countered by the firm's monopoly power.

the two plants.<sup>9</sup> The easiest possible example is as follows: Suppose the product price and total quantity is fixed at  $\bar{x}$  (e.g. by price cap regulation), that production in plant  $i$  is  $x^i = x^i(N_i, K_i)$ ,  $i = 1, 2$ , and that the government have no demand for employment at plant 1 and fixed demand for employment  $\underline{N}$  at plant 2. Moreover, assume that the factors of production are complements, that is,  $x_{NK}^i > 0$ .

Now another feature is added to the problem of time inconsistency: The policy can be changed not only by deviating from the commitment-case investments in the targeted plant 2, but also by deviating from the commitment-case investments in the other plant 1.

If we, as before, assume that the government's only policy instrument is proportional subsidies on labor in the targeted plant, the firm's profit maximization problem is equivalent to the following cost minimization problem:

$$\min_{N_i, K_i} \{w_0 N_1 + w N_2 + r(K_1 + K_2)\} \quad (38)$$

$$\text{s.t. } x^1(N_1, K_1) + x^2(N_2, K_2) \geq \bar{x}. \quad (39)$$

From the first order conditions we get the following expression for the made-to-measure optimal wage level in plant 2:

$$w(K_1, K_2, N_1) = w_0 \frac{x_N^2(\underline{N}, K_2)}{x_N^1(N_1, K_1)} \quad (40)$$

where  $N_1$  satisfies constraint (39) with equality. By differentiation we obtain the following expression for how the optimal wage in plant 2 changes with investments in the two plants. First  $K_1$  (I have omitted the arguments for expositional clarity):

$$\frac{\partial w(K_1, K_2)}{\partial K_1} = w_0 \frac{-x_N^2 \left[ x_{NK}^1 + x_{NN}^1 \frac{\partial N_1}{\partial K_1} \right]}{(x_N^1)^2} = w_0 \frac{-x_N^2 \left[ x_{NK}^1 + x_{NN}^1 \frac{x_K^1}{x_N^1} \right]}{(x_N^1)^2} < 0. \quad (41)$$

( $\frac{\partial N_1}{\partial K_1}$  is found by implicit differentiation of constraint (39).) Next consider  $K_2$ :

$$\frac{\partial w(K_1, K_2)}{\partial K_2} = w_0 x_{NK}^2 > 0, \quad (42)$$

which establish the following proposition:

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<sup>9</sup>Two interesting examples are the Norwegian firms Norcem and Norsk Hydro. The former produces cement in one centrally located plant and one remote located plant, while the latter produces artificial fertilizers with a very similar geographical production structure (see Sørsgard (1989, 1992) and Gabrielsen (1989) for descriptions of the two markets).

**Proposition 8** *If the government wish to subsidise employment in one of the plants in a two-plant price cap regulated monopoly, time inconsistency creates incentives to underinvest in the supported plant, and overinvest in the other; both offsetting the employment-creating effects of the originally proposed subsidy.*

The intuition for this result goes as follows. A second plant can be thought of as a competitor. For a given market size, the subsidies to plant 2 must be large enough to make plant 2 sufficiently competitive relative to plant 1. Investments in plant 1 makes this plant lower this plant's production costs, and as a consequence the government must increase the subsidies to plant 2 in order meet the targeted employment. Note that the firm has incentives to overinvest in plant 1 despite the fact that the actual production in plant one will not change as a result of the investment; the government will always set the subsidy such that it neutralizes the production effects of investing in plant 1.

## 7 Conclusions

What drives the time inconsistency problem is a combination of imperfect policy instruments and adverse sequential structure of the game; the firm can commit to certain actions before the government can commit to a policy.

Throughout the analysis, we have seen that the nature of the problem is sensitive to the structural and behavioural assumptions underlying the game. In particular, we have seen that the problem increases when

- product demand is elastic
- the factors of production are more complementary to each other
- the protected firm has other plants that are not supported

The results can be divided into three groups. First, we have established a link between the two earlier identified causes of time inconsistency. We have shown that increasing the government's elasticity of demand for labor makes strategical underinvestment by the firm less of a problem. However, this is at least partially offset by increasing incentives for the government to act opportunistically. Assuming perfect foresight, the firm will anticipate that the government will reduce subsidies, and respond by reducing its investments.

Second, in stark contrast to Tornell's (1991) results, we find that for some demand conditions, time inconsistency may actually cause overinvestment. This is for instance the case when the government's employment demand is fixed and investments decrease the market value of the marginal product of labor.

Third, we have shown that when the market structure is exogeneously given, optimal employment-enhancing policy will normally involve different wages facing different firms, that is, firm-specific labor subsidies.

Finally, we have shown that time inconsistency may be more of a problem in a two-plant monopoly than in a one-plant monopoly. The intuition is that adding another plant gives the firm another instrument to affect the policy decision.

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