Price discrimination with uncertain consumers

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Abstract

Many goods and services are priced non-linearly, and a common way to implement nonlinear price schedules is to let consumers choose from a menu of two-part tariffs. If consumers know their demand at the time they choose tariff, there is often no principal difference between a fully nonlinear shedule and a (rich enough) menu of two-part tariffs. When consumers do *not* know their exact demand the two are not equivalent, and this paper analyzes the pros and cons of forcing the consumers to choose tariff before they learn their demand.

1 Introduction

For many goods and services that one can buy, the average price depends on the purchased quantity.[1] Such *non-linear* pricing can be defended on welfare grounds as

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[1]Many service providers typically charge a fixed monthly fee in addition to a constant marginal price per unit of service (utilities, telephone companies, ISPs), sometimes one can choose from a menu of tariffs, each tariff intended for a particular subgroup of consumers (transport companies

well as profit considerations. Efficient allocations can sometimes be implemented by setting marginal prices equal to marginal costs, covering any fixed costs by charging a fixed fee. Profit considerations pull in the same direction: profit can sometimes be maximized by first maximizing social surplus (which – again – typically requires marginal price equal to marginal cost) and then transferring this surplus to the firm via a fixed fee. Utilities is one example of services that are priced non-linearly for this reason.

A problem with non-linear pricing is that, typically, neither the price-setting firm nor the social planner know each individual consumer's willingness to pay for a good. Consequently, they do not know how large a fixed fee each individual consumer can bear. Second-degree price discrimination has been suggested as a solution to this problem. In its simplest form, second-degree price discrimination is based on letting the customers choose between two different two-part tariffs, each consisting of a fixed fee k_i and a marginal price p_i. Hence, a consumer with demand q is charged $T_i(q) = k_i + p_i q$ if he chooses tariff i. One of the tariffs is intended for "low-demand" consumers and the other for "high-demand" consumers. The former entails a low fixed fee but a high marginal price, while the latter consists of a high fixed fee and a low marginal price. See Fig. 1 below.

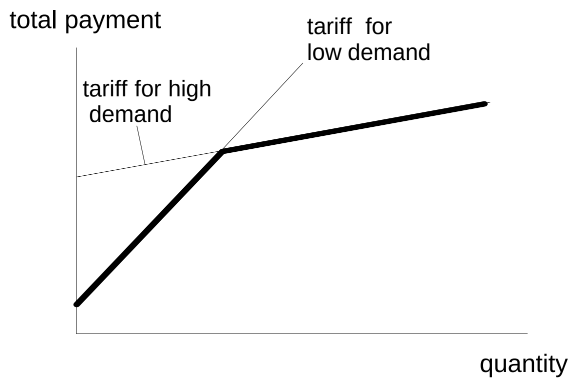

Figure 1: A menu of two two-part tariffs

issue one-way tickets, return tickets, multiple trip tickets as well as monthly tickets; telephone companies have tariffs intended for high-demand and low-demand customers).

Note that the two straight lines in Figure 1 cross, implying that low-demand and high-demand consumers will prefer different contracts. If the consumer knows his future demand (preferences, restrictions, means, etc.) at the time he is given the choice of tariff, he will choose the tariff that minimizes his outlays for the planned demand q^*. This amounts to choosing tariff i if and only if $T_i(q^*) \leq T_j(q^*)$, or, equivalently, that the effective tariff can be conceived of as the bold piece-wise linear function in Fig. 1, that is, the lower envelope of all the payment functions. Given this payment-minimizing lower envelope of tariffs, he should pick the demand that maximizes his utility.

Note that as long as the consumer knows his demand when asked to pick a tariff, neither the consumer nor the firm care about whether the desired pricing scheme is implemented via a menu of two-price tariffs or directly, giving the consumer one single non-linear tariff consisting of the lower envelope of the two-part tariffs in the menu just mentioned (i.e., the bold curve in Fig. 1). This indifference or equivalence may at first glance seem surprising, since real-world firms typically use menus of two-part tariffs.[2] Typically, firms resort to simplified menus of two-part tariffs instead of the fully non-linear tariff, and the most common explanation is that it is difficult for the customers to relate to non-linear schemes, due to limited rationality or rather limited cognition.[3]

The aims of the present paper are twofold. First, I will provide another reason why firms may prefer menus of contracts to non-linear contracts. The equivalence just mentioned breaks down if the consumer has less than perfect information about

[2] Most literature on price discrimination focus on deriving optimal non-linear mechanisms, and treat menus of two-part tariffs only as a way to implement the optimal mechanism. That is, the literature stress the *equivalence* between menus of two-part tariffs and other implementations (the more direct implementation inclusive).

[3] As already indicated, most instances of non-linear pricing is implemented as menus of two-part tariffs. Notable exceptions are the mobile phone companies Sonofon and Cosmote. Sonofon (Denmark) have a calling plan called Variant. (See www.sonofon.dk for detailed information in Danish.) Subscribers get their phone bill computed in three different ways, and they are then charged according the lowest of the three. Also Cosmote (Greece) uses volume discounts in their calling plans. Telia (Sweden) introduced a similar scheme in 1997, but has recently abandoned it.

his future demand at the time he is forced to choose from the menu. If a consumer is uncertain whether his demand will be high or low, he will prefer the contract consisting of the bold curve, as this contract minimizes his outlays for any realized demand. Note that offering the bold curve is equivalent to letting the consumer choose tariff *after* his demand uncertainty has resolved. However, it is by no means clear that the firm have the right incentives to offer the bold curve (or, equivalently, to let the consumer choose tariff ex post): a consumer who is forced to choose tariff before learning his demand will, on average, end up paying more for a given demand that if he had been allowed to choose tariff ex post (or if the cost-minimizing lower envelope had been offered).

From this simple example one can get the impression of having the explanation why firms often force their customers to choose tariff in advance, even though the customers would have preferred to choose ex post: firms exploit the customers' uncertainty to charge them more. Similarly, it is tempting to conclude that the government should interfere, to the extent that excessive payments implies inefficiently low demand. Both these conclusions are a bit hasty, though, since they build on comparing two exogenously given pricing rules. Clearly, the design of the tariffs should depend on whether the consumers have to choose tariff at an early stage or can wait until they know more about their demand. Since the customers will prefer (i.e., they have a higher willingness to pay for) schemes consisting of the lower envelope in the example above, there are good reasons to believe that late choices are tied to higher fixed fees. This would in turn make late choices more attractive for firms and less attractive for consumers, rendering the net effect on consumer utility as well as profit indeterminate at first glance.

I set up a model of price discrimination toward consumers who are uncertain about their own demand at the time they must choose a tariff. The focus is on the endogeneity of information: the seller may affect the information through a requirement of early commitment to a contract (in contrast to no such commitment, with ex post payment minimization as the result). It turns out that the seller prefer to have as little informed consumers as possible at the time they choose contract, not for the reasons just explained, but because more information tend to increase the differences between consumers, and differences make it more difficult to extract the consumers' surplus. As an extreme, if consumers are totally uninformed and all regard themselves as "average" consumers, then the first best is achieved by marginal cost pricing combined with a fixed fee that extracts all expected consumers' surplus, implying that having uninformed consumers choose tariffs is also best from a social point of view.[4]

Unfortunately, it is rarely possible to get your customers to choose from a menu of tariffs before they have learned anything that can distinguish them from other consumers — realistic situations involve consumers who have some imperfect information about their future demand. The second aim of the paper is to design an appropriate mechanism for such situations.

I conclude this introduction with a brief review of some relevant literature. The modern treatment of mechanism design in general and nonlinear pricing in particular started with Mirrlees's (1971) reformulation of the incentive constraints embedded in problems of mechanism design with asymmetric information. Seminal contributions to welfare analyses of nonlinear pricing include Spence (1977), Roberts (1979) and Katz (1983). An excellent survey of nonlinear pricing is Wilson (1993). Important contributions to the closely related theory of mechanism design in regulatory relationships include Baron and Myerson (1981) and Laffont and Tirole (1986), see Laffont and Tirole (1993) for a comprehensive survey.

The other vein of literature concerns agency problems with imperfectly informed agents. A good starting point is Lewis and Sappington (1993). Kessler (1998) studies situations in which imperfect information may be good for the agent (in contrast to what I find in the present model), while Lewis and Sappington (1997) and Cremer, Khalil and Rochet (1998a, 1998b) study agents' incentives to engage in information acquisition before or after a contract is signed.

The paper proceeds as follows. The model is laid out in the next section. The case of totally uninformed consumers is studied in Section 3, the properties of the

[4]If fixed costs are small, sometimes a social planner can achieve first best also without forcing consumers to choose tariffs before they become informed, but in general, late choices imply heterogeneous consumers which then implies distortionary pricing and welfare losses.

optimal ex-ante menu of two-part tariffs with imperfectly informed consumers is first derived for rather general functional forms in Section 4, while an example is examined in more details in Section 5. Some concluding remarks are gathered in Section 6.

2 Model

Consider a monopolist setting prices in a market with a continuum of consumers. Consumer preferences are described by utility functions that are linear in money and non-linear in quantity demanded:

$$u(q, \theta, T) = V(q, \theta) - T \tag{1}$$

where q is quantity consumed, θ is a demand parameter and T is the payment charged. The demand parameter θ is distributed over an interval $[L, H]$ according to a cumulative distribution function $F(\theta)$. Let $f(\theta)$ denote the corresponding density function, and let $c \in [0, L]$ denote marginal production costs. It will be assumed that the marginal utility of consumption is positive and decreasing (letting subscripts denote partial derivatives, this means that $V_q > 0$ and $V_{qq} < 0$), that utilities are increasing in the demand parameter ($V_\theta > 0$), and that the single-crossing condition is satisfied ($V_{q\theta} > 0$). Moreover, I assume that the distribution function satisfies the monotonous hazard rate property: $\frac{f(\theta)}{1-F(\theta)}$ increases with θ.

The timing of decisions and events is as follows:

1. Each consumer receives a signal s of their demand parameter.

2. The firm decides whether to force the consumers to choose a contract *now* or *later*. In the former case, a mechanism is designed and the consumers act on the mechanism in order to maximize expected utility (expectation taken over demand parameters) given their information s.

3. Each consumer learns the true value of his demand parameter θ.

4. If a mechanism has not been designed in Step 2 above, then the firm now designs a mechanism and the consumers act on the mechanism in order to maximize utility given their information θ.

5. The mechanism from Step 2 or 4 is executed. (A quantity is purchased and paid for according to the chosen tariff.)

There are many ways to model the notion of imperfectly informed consumers, that is, the way the signal s relates to the true demand parameter θ. The following way turns out to be convenient, as it allows us to obtain closed-form solutions to the game for reasonable parameterizations. First, let the signal s be drawn from the same distribution as is the demand parameter, i.e., from F. Second, with probability α the signal equals the true demand parameter. With the complementary probability $1-\alpha$, however, the signal contains no information, in which case the true demand parameter is independently redrawn from the same distribution F. The consumer does not know whether the signal is correct or not until stage 3. In what follows the parameter α will be referred to as the *quality* of the signal, for quite obvious reasons: if $\alpha = 0$ the signal contains no information; if $\alpha = 1$ the signal gives the consumer perfect information. For in-between values of α the consumer has some, but not perfect information, and the higher α the more information he has.[5]

Both the signal s and, eventually, the true value θ of the demand parameter are private information for each consumer, while the information technology (α inclusive) is common knowledge.

When it comes to pricing, I assume that the monopolist designs a menu consisting of a continuum of two-part tariffs, whether this is done at stage 2 or 4.[6]

[5] This information technology was developed by Erbenová and Vagstad (1999).

[6] This is without loss of generality when the consumers know their demand parameters before choosing (since it then turns out that the general non-linear mechanism is implementable in two-part tariffs), but probably less innocent when they do not, and in this respect the present analysis should be considered a first step. One defense of this first step is that it is in line with what firms actually do when constructing tariffs among which uninformed consumers can choose, but this is of course not a satisfactory argument from a theoretic point of view.

3 Preliminaries: full or no information

In the introduction I claimed that if consumers have no information to distinguish themselves from average consumers, then first best can be achieved. It is now time to give this claim a more precise formulation:

Proposition 1 *If $\alpha = 0$, then the first best can be implemented by offering the following single two-part tariff to all consumers: $p_i = p = c$ and*

$$k_i = k = \int_L^H \left(V(q^*(\theta), \theta) - cq^*(\theta)\right) dF(\theta)$$

where $q^*(\theta)$ denotes the first-best demand corresponding to prices equal to marginal costs.

The interpretation is straightforward: marginal cost pricing maximizes social surplus, and this particular fixed fee extracts all consumers' surplus. With no information ex ante, every consumer face the same distribution of demand parameters and have therefore the same willingness to pay for a tariff involving marginal cost pricing.

Next, consider the other extreme, that is, the consumers know their demand parameters when they make the decision whether or not to accept a contract offered by the monopolist. The problem facing the monopolist is then a standard mechanism design problem, as laid out in most modern IO textbooks. The following exposition follows that of Tirole (1988, section 3.5.1.2). A solution of the non-linear pricing problem is there described as a type-quantity schedule $q(\theta)$ with a corresponding payment $T(q(\theta))$, and given the assumptions of single crossing and monotonous hazard rate this tariff is implementable by a continuous menu of two-part tariffs. To be more precise, the monopolist maximizes

$$\pi_m = \int_L^H \left(T(q(\theta)) - cq(\theta)\right) dF(\theta) \tag{2}$$

subject to the following familiar participation and incentive constraints:

$$V(q(\theta), \theta) - T(q(\theta)) \geq 0 \text{ for } \theta \geq \hat{\theta} \tag{3}$$

$$V(q(\theta), \theta) - T(q(\theta)) \geq V(q(\theta'), \theta) - T(q(\theta')) \text{ for all } \theta, \theta' \tag{4}$$

where $\hat{\theta}$ is a cut-off demand parameter (possibly equal to the lowest type possible — $\theta = L$) with the property that consumers with $\theta < \hat{\theta}$ are not served. These constraints are not very tractable. Fortunately, among the participation constraints it suffices to include the constraint for the lowest type participating. Moreover, it suffices to have the incentive constraints being satisfied locally. Hence the constraints can be written (see Tirole, 1988, for details)

$$V(q(\hat{\theta}), \hat{\theta}) - T(q(\hat{\theta})) = 0 \tag{5}$$

$$\frac{\partial V(q(\theta), \theta)}{\partial q(\theta)} = T'(q(\theta)) \tag{6}$$

Maximizing profit subject to these two constraints can now be shown to yield a type-quantity schedule $q(\theta)$ satisfying

$$\frac{\partial V(q(\theta), \theta)}{\partial q(\theta)} = c + \frac{1 - F(\theta)}{f(\theta)} \frac{\partial^2 V(q(\theta), \theta)}{\partial q(\theta) \partial \theta} \tag{7}$$

for θ higher than or equal to a cutoff type $\hat{\theta}$. Note in particular that the last term on the right hand side of eq. (7) is positive, implying that the optimal mechanism entails a markup over marginal costs, i.e., a downward distortion of quantity.[7]

If welfare is defined as the sum of producer's and consumers' surplus, the welfare comparison of the two cases is straightforward: the first case describes a first-best situation, while the second entails distortions. From the firm's perspective there is yet another reason to prefer the uninformed consumers: not only is there no distortions, but also no rent (in expectation) left to the consumers: all the value created ends up in the firm. This yields

Proposition 2 *When $\alpha = 0$, ex ante commitment to a menu of tariffs is always better than no such commitment, both for the monopolist and for society.*

Clearly, as consumers are different *ex post*, the fact that their average rent equals zero means that some consumers — those who draw "low" demand parameters —

[7] Given the type-quantity schedule $q(\theta)$, the optimal menu of two-part tariffs is now easily found. First, marginal price must equal marginal utility, and this yields a type-price schedule $p(\theta)$. Second, from (5) and (6) we can compute a type-payment schedule $T(\theta)$ and the corresponding fixed fee $k(\theta) = T(\theta) - p(\theta)q(\theta)$. Eliminating θ yields the corresponding price-fee schedule $k(p)$.

end up with negative utility, thereby regretting entered into the relationship with the monopolist in the first place. But at the time when they learn this, it is too late, as they will already have accepted the proposed tariff, and the too high fixed fee is then already a sunk cost.

4 Intermediate cases

We now move on to the more realistic cases in which the firm does not face consumers that are totally ignorant about their own characteristics at the time they meet the firm for the first time, but who already have some information that distinguishes them from the crowd. That is, situations with $\alpha \in (0, 1)$. Two questions then arise. First, for these intermediate cases, is it still true that committing to a menu of two-part tariffs while the consumers still has imperfect information is better than no such commitment? Second, if the principal should choose to offer a mechanism to the imperfectly informed consumers, what does the optimal mechanism look like?

We start with the first question. It is not absolutely clear that early commitment is the better choice also for intermediate values of α. What is attractive with having totally uninformed consumers is partly that it fully separates rent extraction from efficiency considerations. When consumers have some imprecise information ex ante there will be inefficient rent extraction also with commitment to a menu of two-part tariffs, and as a consequence the firm will impose price discrimination entailing distortions of quantities. But because the information is imprecise, ex post it will turn out that some high-demand consumers have chosen the very inefficient low-demand contract, while some low-demand consumers will choose the more efficient high-demand contracts. But the pros and cons of these mistakes do not necessarily cancel out, since it is often more important to have small distortions for high-demand consumers.

Moving to the second question, I start by restricting attention to mechanisms consisting of menus of two-part tariffs. This is not the only way to proceed from this point, but a natural first step, cf. the discussion in the introduction. The monopolist offers the consumers a choice from a menu of two-part tariffs $\{p, k(p)\}$, where a consumer who chooses a marginal price of p has to pay a fixed fee of $k(p)$. If this consumer purchases q units of the good in question, his payment is given by

$$T = k(p) + qp \tag{8}$$

Clearly, p and k must be inversely related — a low marginal price must be accompanied by a high fixed fee, and vice versa. Choosing a tariff is in this setting equivalent to choosing p. Each type — now characterized by his signal s — choose the contract $p(s)$ that maximizes his expected utility, given the signal.

Lemma 3 *The expected utility of a consumer with signal s choosing tariff p can be written*

$$\begin{aligned} U(s) &= \alpha \left(V\left(q^*(s,p),s\right) - pq^*(s,p)\right) \\ &\quad + (1-\alpha)\int_L^H \left(V(q^*(\theta,p),s) - pq^*(\theta,p)\right) dF(\theta) - k(p) \end{aligned}$$

where $q^*(\theta,p) = \arg\max_q \{V(q,\theta) - pq\}$.

Proof: There are two cases to consider. First suppose the signal is correct, and the consumer receives utility (gross of the fixed fee) of $V\left(q^*(s,p),s\right) - pq^*(s,p)$. This happens with probability α. Next we look at the remaining cases, in which the signal contained no information, and the demand parameter is redrawn from the original distribution, yielding gross expected utility of $\int_L^H \left(V(q^*(\theta,p),\theta) - pq^*(\theta,p)\right) f(\theta)d\theta$. This happens with probability $1-\alpha$. ¥

Then we take a closer look at which constraints it is natural to impose on the monopolist's problem, that is, which incentive and participation constraints should be in place. Following the steps of Tirole (1988), it is easily verified that the following constraint is necessary and sufficient to make a consumer having received a signal s choose his designated tariff $p(s)$:

$$U'(s) = \alpha \frac{\partial \left(V\left(q^*(s,p(s)),s\right)\right)}{\partial s} \tag{9}$$

This is the imperfectly informed consumers version of the corresponding incentive constraint (6) from the previous section. Next we find an expression for expected profit from a consumer with a given signal, take expectation over all possible signals, and maximize this expectation with respect to the menu of linear tariffs. To simplify notation, in what follows we will assume that marginal costs equal zero. The firm's profit can then be expressed as

$$\begin{aligned}\pi &= \int_L^H \left(k(p(s)) + (p(s)-c)\left(\alpha q^*(s,p(s)) + (1-\alpha)\int_L^H q^*(\theta,p(s))dF(\theta)\right)\right) dF(s) \qquad (10)\\ &= \int_L^H \left(\alpha V\left(q^*(s,p(s)),s\right) + (1-\alpha)\int_L^H V(q^*(\theta,p(s)),\theta)dF(\theta) - U(s)\right) dF(s)\end{aligned}$$

Integrating by parts, the last term can be rewritten:

$$\begin{aligned}\int_L^H U(s)dF(s) &= \left[U(s)\left(F(s)-1\right)\right]_L^H + \int_L^H (1-F(s))U'(s)ds \qquad (11)\\ &= \alpha \int_L^H (1-F(s))\frac{\partial\left(V\left(q^*(s,p(s)),s\right)\right)}{\partial s}ds\end{aligned}$$

using $U'(s) = \alpha\frac{\partial(V(q^*(s,p),s))}{\partial s}$ (from the incentive constraint), $F(H) = 1$ and $U(L) = 0$ (from the participation constraint). Then expected profit can be written

$$\begin{aligned}\pi &= \int_L^H \left(\alpha V\left(q^*(s,p(s)),s\right) + (1-\alpha)\int_L^H V(q^*(\theta,p(s)),\theta)dF(\theta) - U(s)\right) dF(s)\\ &= \int_L^H \left\{\left(\alpha V\left(q^*(s,p(s)),s\right) + (1-\alpha)\int_L^H V(q^*(\theta,p(s)),\theta)dF(\theta)\right) f(s) \qquad (12)\right.\\ &\quad \left. -\alpha\frac{\partial V\left(q^*(s,p(s)),s\right)}{\partial s}(1-F(s))\right\} ds\end{aligned}$$

This integral is maximized by maximizing the integrand for each value of the signal s. The first order condition reads (suppressing the price arguments of the q^* functions for simplicity):

$$\begin{aligned}&\left(\alpha\frac{\partial V\left(q^*(s),s\right)}{\partial q^*(s)}\frac{\partial q^*(s)}{\partial p} + (1-\alpha)\int_L^H \frac{\partial V\left(q^*(\theta),\theta\right)}{\partial q^*(\theta)}\frac{\partial q^*(\theta)}{\partial p}dF(\theta)\right) f(s) \qquad (13)\\ &= \alpha\frac{\partial^2 V\left(q^*(s),s\right)}{\partial q^*(s)\partial s}\frac{\partial q^*(s)}{\partial p}(1-F(s))\end{aligned}$$

or

$$\frac{\partial V\left(q^*(s),s\right)}{\partial q^*(s)} + \frac{1-\alpha}{\alpha}\int_L^H \frac{\partial V\left(q^*(\theta),\theta\right)}{\partial q^*}\left(\frac{\partial q^*(\theta)}{\partial p}\Big/\frac{\partial q^*(s)}{\partial p}\right) dF(\theta)$$

$$= \quad \frac{1-F(s)}{f(s)} \frac{\partial^2 V\left(q^*(s), s\right)}{\partial q^*(s) \partial s} \tag{14}$$

This expression can now be compared with the corresponding expression for fully informed consumers (eq. (7)):

$$\frac{\partial V(q(\theta),\theta)}{\partial q(\theta)} = \frac{1-F(\theta)}{f(\theta)} \frac{\partial^2 V(q(\theta),\theta)}{\partial q(\theta)\partial\theta} \tag{7}$$

As $\frac{1-\alpha}{\alpha}$, $\frac{\partial V(q^*(\theta),\theta)}{\partial q^*}$ and $\left(\frac{\partial q^*(\theta)}{\partial p} \Big/ \frac{\partial q^*(s)}{\partial p}\right)$ are all positive, the schedule for imperfectly informed consumers features a smaller term $\frac{\partial V(q^*(s),s)}{\partial q^*(s)}$ than does the schedule for their perfectly informed counterparts. Concavity of V then yields the following Proposition:

Proposition 4 *Imperfect information give rise to less distortions (higher q or lower p) for each given type:*

$$\frac{\partial V\left(q^*(s), s\right)}{\partial q^*(s)} < \frac{\partial V(q(s), s)}{\partial q(s)} \Longleftrightarrow q^*(s) > q(s)$$

However, this does not necessarily mean less distortions for each realized demand parameter θ. Further results requires more structure, and in the next section we will investigate an example in some more detail.

5 Example

In this section we will make the following assumptions. First, suppose F is the uniform distribution on $[0,1]$. Second, suppose that $V(q,\theta) = \theta q - \frac{1}{2}q^2$. Then $f(\theta) = 1$, $F(\theta) = \theta$, $q^*(\theta,p) = \arg\max_q\{V(q,\theta) - pq\} = \max\{0, \theta - p\}$. The first order condition for imperfect information case then reads (if $s \geq p$)

$$\alpha p + (1-\alpha)\int_p^1 p d\theta = \alpha p + (1-\alpha)p(1-p) = \alpha(1-s) \tag{15}$$

This second degree equation in p has two roots, the one that makes economic sense can be written

$$p(\alpha, s) = \frac{1}{1-\alpha}\left(\frac{1}{2} - \sqrt{\frac{1}{4} + \alpha s - \alpha - \alpha^2 s + \alpha^2}\right) \tag{16}$$

for $p \leq s$, that is, for

$$s \geq \hat{s}(\alpha) \equiv \frac{1}{2(1-\alpha)}\left(\alpha + 1 - \sqrt{5\alpha^2 - 2\alpha + 1}\right) \tag{17}$$

There will be bunching for s smaller than this value, implying that the pricing schedule can be written[8]

$$p(\alpha, s) = \begin{cases} \frac{1}{1-\alpha}\left(\frac{1}{2} - \sqrt{\frac{1}{4} + \alpha s - \alpha - \alpha^2 s + \alpha^2}\right) & \text{if} \quad s \geq \hat{s}(\alpha) \\ \frac{1}{2(1-\alpha)}\left(\alpha + 1 - \sqrt{5\alpha^2 - 2\alpha + 1}\right) & \text{if} \quad s < \hat{s}(\alpha) \end{cases} \tag{18}$$

Below we have plotted pricing schedules corresponding to different values of α.

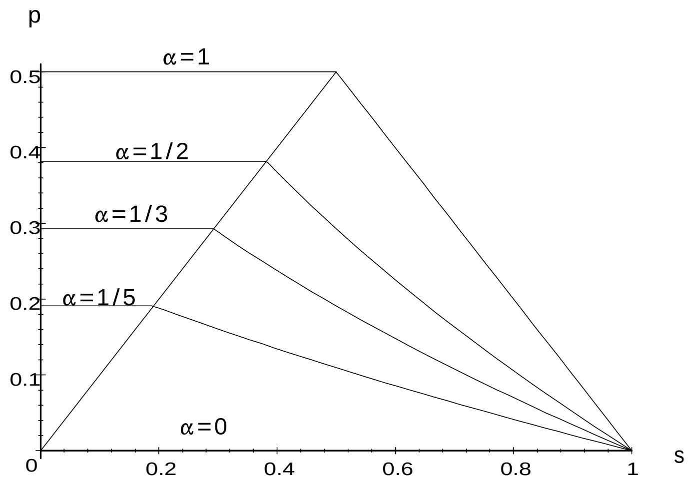

Figure 2: Optimal price as function of type s for different α

Note in particular that the polar cases emerge as special cases here: If $\alpha = 0$, all types face a marginal price of zero, while if $\alpha = 1$, the Tirole scheme involving marginal prices as a non-increasing function of type is implemented. Perhaps not very surprising, pricing is a continuous and monotone function of signal quality, implying that the distortions are increasing in signal quality α.

Once the marginal pricing schedule is established, one can find the remaining part of the pricing schedule, the fixed fee $k(p)$, as well as other aspects and consequences of the optimal pricing rule. Details are found in the appendix, here we will only

[8] Bunching will occur here because for sufficiently low signal the customer's only hope for a surplus stems from the possibility that the signal is wrong, in which case the original signal does not matter for the customer's valuation of the contract.

show and discuss some graphs.

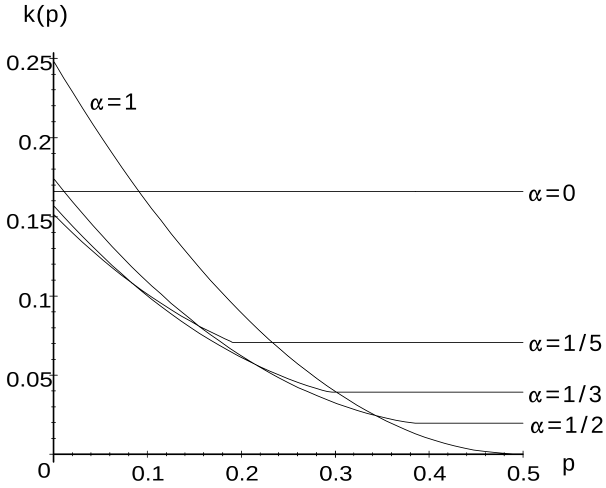

Figure 3: Combinations of prices p and fees k for different α

Fig. 3 shows how the optimal mechanism $k(p)$ looks like for different signal qualities.[9] The perhaps most obvious pattern that can be seen is the negative relationship between marginal prices and fixed fees. Perhaps less obvious is the fact that as α increases from 0, consumers will be facing schedules involving an option with lower fixed fees yet zero marginal price. (Clearly, this only applies for small α.) This only reflects that while uninformed consumers earn zero rent, informed (also imperfectly informed) consumers earn strictly positive rent. Finally, note that as information improves, the schedules become steeper.

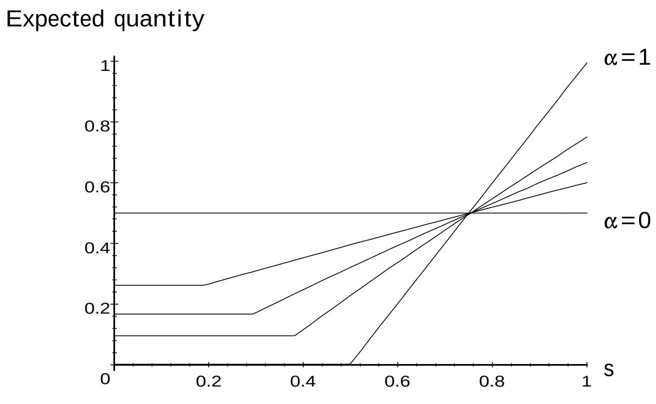

Figure 4: Expected quantity for different types s and α

[9]Note that whenever a graph is horizontal, the customer will "move to the left", since a lower marginal price can be chosen at the same fixed fee. Consequently, the horizontal segments are superfluous as they will never attract any customers.

Figure 4 shows expected quantity as a function of signal s for different signal qualities α. It is easily verified that expected quantity (expectation taken over all consumer types) decreases as information improves.

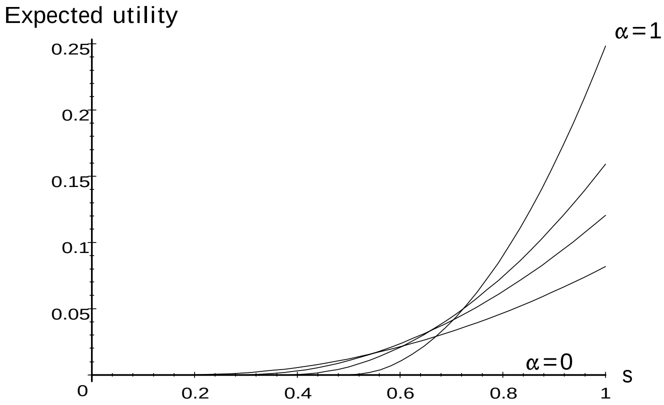

Figure 5: Expected utility for different types

Similarly, Figure 5 reveals that while the average customer benefits from improved information, some consumers do not. The reason is basically the same as in Kessler's (1998) work on the value of information: *ceteris paribus*, (more) information is good, but the fact that more informed consumers make the firm impose more rent-extracting distortions may reverse this result.

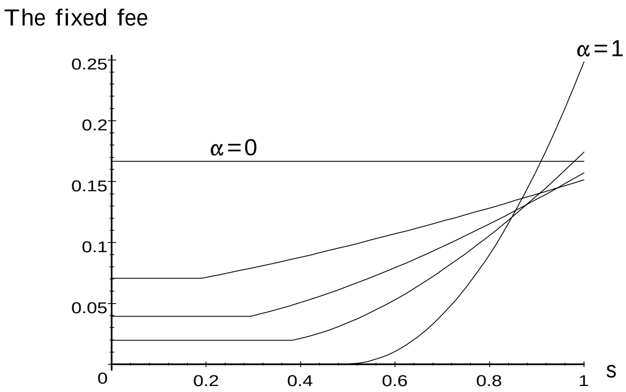

Figure 6: The fixed fee for different types

Figure 6 shows shows how the type-fee mapping changes with signal quality. Again the most pronounced pattern is the increased steepness following increased signal quality. Figure 7 below shows a similar pattern, now dealing with type-dependent profit levels.

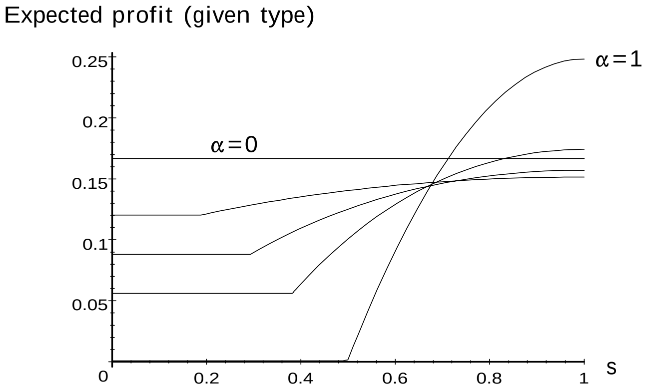

Figure 7: Type-dependent profit levels

Finally, and perhaps most interesting, Figure 8 reveals that profit and welfare are monotonously decreasing functions of signal quality, while consumers' surplus is a monotonously increasing function of signal quality:

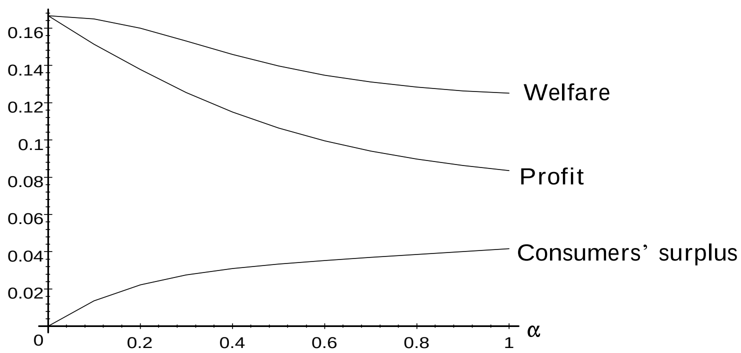

Figure 8: Expected profit, consumers' surplus and welfare

This implies that at least for this parameterization, a variant of Proposition 2 holds for intermediate signal quality: Also when the consumers have some private information at the time they can be offered a contract, any existing uncertainty can be exploited to the benefit of the firm as well for the society — it is still the case that early commitment to a menu of two-part tariffs is better than no such commitment.

6 Concluding remarks

The aim of this paper has been to shed light on the differences between a direct implementation of a non-linear price schedule on the one hand and implementation using a menu of two-part tariffs on the other. If the consumers have imperfect information about their demand when the choice of tariff is made, then using a menu of two-part tariffs lead to higher profits and welfare but lower consumer utility.

While received theory can tell us how the optimal menu of two-part tariff works for perfectly informed consumers, this has not been done before for imperfectly informed consumers. I have derived the optimality conditions for the general problem, and studied in detail the solution of a specific example involving uniform distributions and quadratic utility. The resulting tariff schedule resembles the corresponding schedule for fully informed consumers. However, information about the ex post spread of demand is insufficient to construct the optimal menu for imperfectly informed consumers.

The practical implications of the analysis are twofold. First, monopolists should ask their customers to choose from a menu of tariffs instead of offering them a single non-linear contract. The point is to enter a binding agreement with the customers as early as possible, i.e., before they receive too precise private information about own demand. This is a rather general point, and should therefore not be confined to uncertainty about own demand volume, but apply to other types of consumer uncertainty too. Second, the government should not interfere against such pricing, as this is also good for welfare.

Several matters of importance has been left out of the analysis. Most importantly, the analysis is confined to monopoly pricing, and a natural next step is to extend the analysis to competitive environments. Although we cannot know for sure, I suspect that since using menus of two-part tariffs is not only profit maximizing but also more efficient than the corresponding single non-linear tariff, they might survive competition. However, it should be clear that competition would shift much of the efficiency gain over to the consumers.

As noted in the introduction, truly non-linear tariffs are a rare sight compared to the relative abundance of menus of two-part tariffs. Therefore, the few examples are worth a closer look. The mobile phone operators Sonofon and Cosmote have replaced menus by non-linear tariffs. This replacement has been regarded successful in terms of market shares, a consideration that does not arise in a monopoly model. One explanation behind the success of Sonofon's non-linear calling plan "Variant" is that consumers appreciate a plan that relieves them from the tedious problem of finding the optimal tariff, and this "no fuss"-image has proven easy to market. (Similar views have been expressed concerning Cosmote's non-linear plans.) An interesting case is that of Telia, who had a non-linear plan from 1997 until last year when it was abandoned. One pronounced explanation why it was abandoned was that in an unstable market with an increasing number of competitors the one single plan entails too little flexibility: with a menu of two-part tariffs, prices in one segment of the market can be adjusted according to competition without affecting pricing in all other segments.

I have already indicated that by extending the analysis to situations with some sort of competition, the results would be more widely applicable. The analysis could also be extended in other directions. In particular, the information structure could be modelled in different ways. One could allow the consumers to have different degrees of information at the time the contract is offered (i.e. different α): One extreme example of this would be situations in which consumers know whether their signal is correct or not (as in Lewis and Sappington (1993)).

7 Appendix: Example details

This appendix presents the calculus behind the graphs in the main body of the paper. To recapitulate, it is assumed that $F(\theta) = \theta$ for $\theta \in [0, 1]$, and that $V(q, \theta) = \theta q - \frac{1}{2}q^2$. Then $f(\theta) = 1$, $F(\theta) = \theta$, and $q^*(\theta, p) = \arg\max_q \{V(q, \theta) - pq\} = \max\{0, \theta - p\}$. The optimal ex ante mechanism then features

$$p(\alpha, s) = \begin{cases} p_2(\alpha, s) & \text{if} \quad s \geq \hat{s}(\alpha) \\ p_1(\alpha) & \text{if} \quad s < \hat{s}(\alpha) \end{cases} \tag{19}$$

where

$$p_1(\alpha) = \frac{1}{2(1-\alpha)} \left(\alpha + 1 - \sqrt{5\alpha^2 - 2\alpha + 1}\right) \tag{20}$$

$$p_2(\alpha, s) = \frac{1}{1-\alpha} \left(\frac{1}{2} - \sqrt{\frac{1}{4} + \alpha s - \alpha - \alpha^2 s + \alpha^2}\right) \tag{21}$$

$$\hat{s}(\alpha) = \frac{1}{2(1-\alpha)} \left(\alpha + 1 - \sqrt{5\alpha^2 - 2\alpha + 1}\right) \tag{22}$$

Next, integrating the incentive constraint yields the following type-utility schedule, i.e., the expected utility for an agent with signal s:

$$u(\alpha, s) = \begin{cases} \int_{\hat{s}(\alpha)}^{s} \alpha(t - p(\alpha, t))dt & \text{if} \quad s \geq \hat{s}(\alpha) \\ 0 & \text{if} \quad s < \hat{s}(\alpha) \end{cases} \tag{23}$$

Moreover, expected demand (quantity) of an agent with signal s can be written

$$q(\alpha, s) = \begin{cases} q_2(\alpha, s) & \text{if} \quad s \geq \hat{s}(\alpha) \\ q_1(\alpha) & \text{if} \quad s < \hat{s}(\alpha) \end{cases} \tag{24}$$

where $q_1(\alpha) = (1 - \alpha) \int_{p_1(\alpha)}^{1} (t - p_1(\alpha))\, dt = \frac{1}{2} (1 - \alpha) (1 - p_1(\alpha))^2$ and $q_2(\alpha, s) = \alpha (s - p_2(\alpha, s)) + \frac{1}{2} (1 - \alpha) (1 - p_2(\alpha, s))^2$.

Next, the fixed fee for an agent with signal s can now be written

$$k(\alpha, s) = \begin{cases} k_2(\alpha, s) & \text{if} \quad s \geq \hat{s}(\alpha) \\ k_1(\alpha) & \text{if} \quad s < \hat{s}(\alpha) \end{cases} \tag{25}$$

where $k_1(\alpha) = \frac{1-\alpha}{6}(1 - p_1(\alpha))^3$ and $k_2(\alpha, s) = \frac{\alpha}{2}(s - p_2(\alpha, s))^2 + \frac{1-\alpha}{6}(1 - p_2(\alpha, s))^3 - u_2(\alpha, s)$.

To find the price-fee schedule we may for instance invert the type-price schedule to obtain

$$s(\alpha, p) = (1 - p)(1 + p - \frac{p}{\alpha}) \tag{26}$$

which applies for $p < \hat{s}(\alpha)$. Now the price-fee schedule can be expressed as

$$\tilde{k}(\alpha, p) = \begin{cases} k_2(\alpha, s(\alpha, p)) & \text{if} \quad p \leq \hat{s}(\alpha) \\ k_1(\alpha) & \text{if} \quad p > \hat{s}(\alpha) \end{cases} \tag{27}$$

Furthermore, the equilibrium profit the firm makes from an agent with signal s equals

$$\pi_0(\alpha, s) = \begin{cases} \pi_2(\alpha, s) & \text{if} \quad s \geq \hat{s}(\alpha) \\ \pi_1(\alpha) & \text{if} \quad s < \hat{s}(\alpha) \end{cases} \tag{28}$$

where $\pi_1(\alpha) = k_1(\alpha) + p_1(\alpha)q_1(\alpha)$ and $\pi_2(\alpha, s) = k_2(\alpha, s) + p_2(\alpha, s)q_2(\alpha, s)$. Taking expectation over all possible values of s then yields equilibrium expected profit Π, consumer's surplus U and their sum welfare W as functions of α alone:

$$\Pi_0(\alpha) = \int_0^1 \pi_0(\alpha, s) ds \tag{29}$$

$$U(\alpha) = \int_0^1 u(\alpha, s) ds \tag{30}$$

$$W(\alpha) = \Pi_0(\alpha) + U(\alpha) \tag{31}$$

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