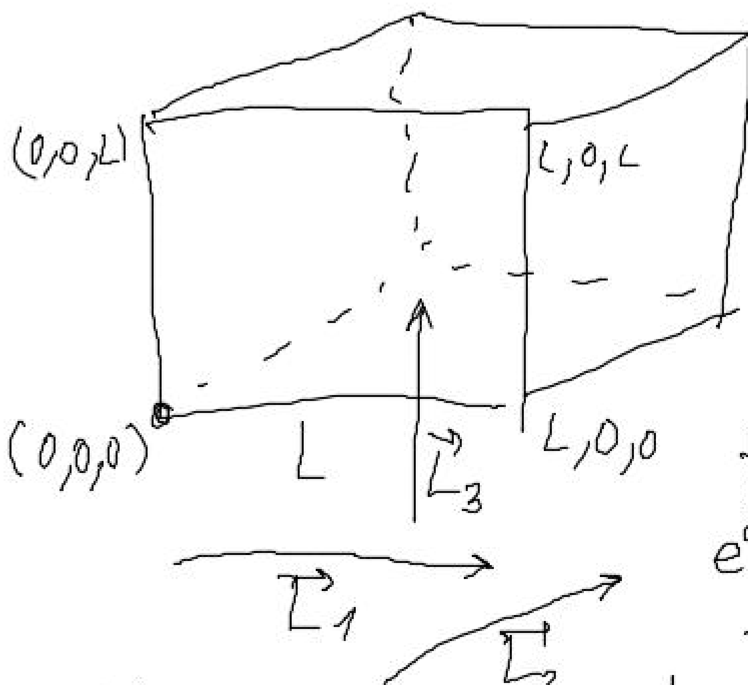
Lecture Thursday 25. October 2007

Topics: Density of States, Closing the Light-atom Interaction

Comment: The density of states treatment presented here is relevant for many fields including particle emission (e.g. autoionization) as well as discretization (introduction to Fourier integrals)



$V = L^3$ ~~infinitely hard $\psi(L) \to 0$~~

Periodic boundary conditions

$\psi(0) = \psi(L, \ldots)$

(from wall to wall ...)

free inside $\approx \underline{\underline{e^{i\vec{k}\cdot\vec{r}}}}$

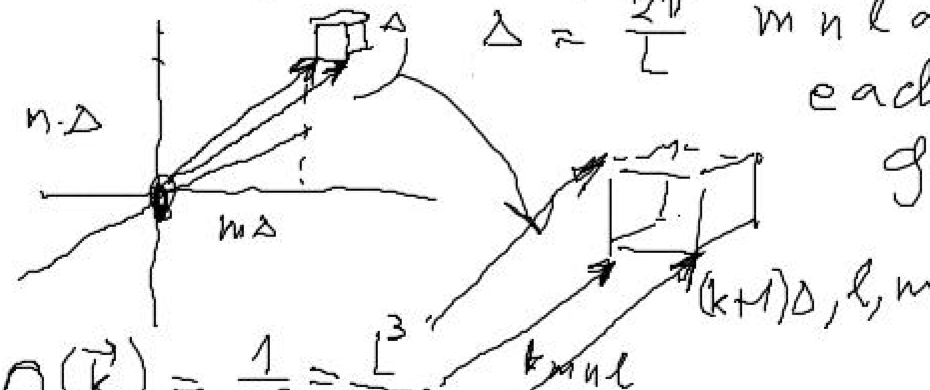
$e^{i\vec{k}\cdot\vec{L}_\alpha} = 1$

$\longrightarrow$ allowed values of $\underline{\underline{\vec{k}}}$

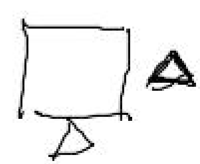
it is easy to find $\vec{k}_{mnl} = (m\Delta, n\Delta, l\Delta)$

$\Delta = \frac{2\pi}{L}$ m n l are integers

each of the vectors gets one little box



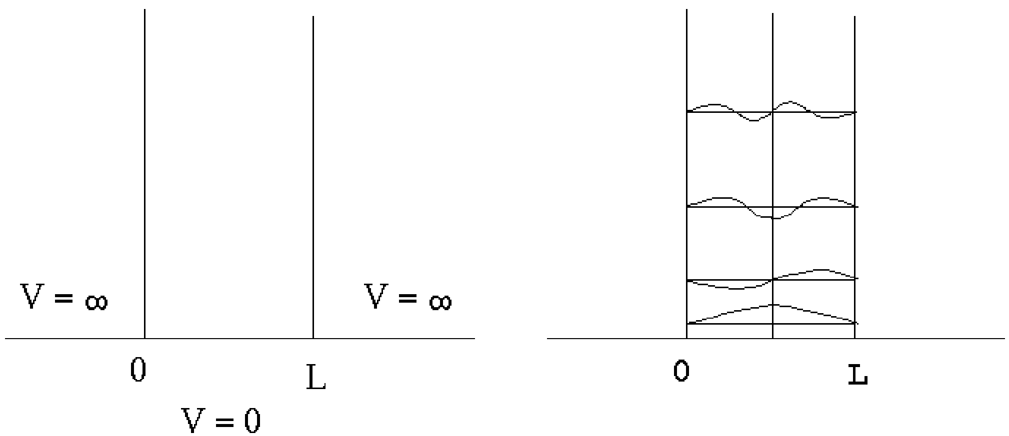
$\rho(\vec{k}) = \frac{1}{\Delta^3} = \frac{L^3}{(2\pi)^3}$



Density of States: we assume that the processes are happening immersed in a large box of edge-length L and volume $V=L^3$

Periodic boundary conditions:

This is a sort of trick which reflects the necessity to obtain «traveling waves. Any «closed box» would lead to “standing waves”, as e.g. in simple treatment of one-dimensional infinite heigth potential well



The periodic boundary conditions leaves the lowest state unchanged, but then pairs all the sin and cosine following solutions to the same energy – pairwise, so that we get a possibility to construct the traveling solutions

cos kx, sin kx have the same energy;

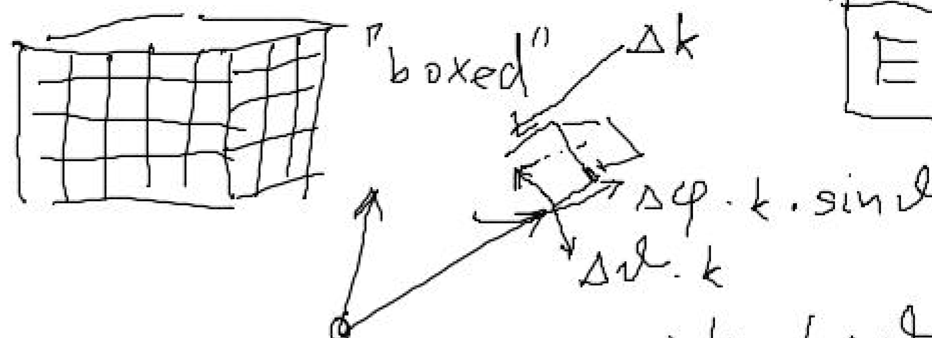
the pair of solutions (cos kx , sin kx) can be combined to a pair (e^{ikx} , e^{-ikx})

$E = ?$ → particles $\frac{\hbar^2 k^2}{2m} = E$ .....

→ photons (field eigenmodes) $\omega \quad \hbar\omega = E$

$\omega = k \cdot c$

$$E = \hbar k c$$



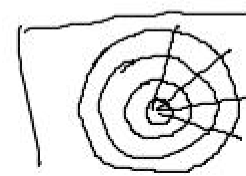
$\Delta k \cdot k\Delta\vartheta \cdot k \cdot \sin\vartheta\, \Delta\varphi$

$k^2 \cdot \Delta k \cdot \underbrace{(\sin\vartheta\, \Delta\varphi) \cdot \Delta\vartheta}_{\Delta\Omega}$

solid angle Ω

to cover a unit sphere

→ sum over "all" $\Delta\Omega$



angle

Δk (radial)

$\Delta\varphi$ (circle)

We see easily that the Δk must be as indicated, $\Delta k = 2\pi/L$
Thus each solution has neighbouring solutions displaced by Delta along each axis, so that there is no other allowed solution inside of a little cube with edge Δk

Thus the density in k-space is one solution per $(\Delta k)^3$ i.e. density = $(\Delta k)^{-3}$

Further, when we wish to integrate over the whole space, we go from boxes in Cartesian coordinates to spherical evaluation.

THIS DOES NOT PRESERVE THE SOLUTIONS, i.e. each in its own box;
but now we have already established the number of states and we are
MERELY EVALUATING the numbers.

The Solid Angle $\Delta\Omega$ consists of approximate squares covering the whole sphere 4π .

The somewhat difficult notation follows from the fact that if there is angular dependence, the integral over sphere can not be done until final evaluation. Thus we have along the process the density of states PER ENERGY and PER SOLID ANGLE (of emission)

The rest should be hopefully undestood from the slides-document