

Multipole expansion

Part 2

$$\frac{1}{|\vec{r}_1 - \vec{r}_2|} = \sum_{LM} \frac{4\pi}{2L+1} \frac{r_{<}^L}{r_{>}^{L+1}} Y_{LM}^*(\hat{r}_1) Y_{LM}(\hat{r}_2) \quad (1)$$

where

$$\begin{aligned} r_{<} &= r_1, & r_{>} &= r_2 & \text{for } |\vec{r}_1| < |\vec{r}_2| \\ r_{<} &= r_2, & r_{>} &= r_1 & \text{for } |\vec{r}_1| > |\vec{r}_2| \end{aligned}$$