PHYS 261

Physical Optics

Lecture Notes–Review Questions

Jakob J. Stamnes

Department of Physics and Technology, University of Bergen, Norway
Tel: +47 55 58 28 18. Fax: +47 55 58 94 40. E-mail: JakobJ.Stamnes@ift.uib.no

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0.1 Topic 1: Elementary Solutions of Maxwell's Equations and the Scalar Wave Equation

- Write down Maxwell's equations and explain the meaning of the various field quantities and parameters.
- What are the material equations or constitutive relations for a linear, isotropic medium?
- Derive the continuity equation from Maxwell's equation and explain its physical significance.
- What boundary conditions must be fulfilled at a boundary between two different dielectrics?
- Explain the difference between a dispersive and a non-dispersive medium, and define the phase velocity and the group vekocity. Give an example of a non-dispersive medium.
- At which of these two velocities will a narrow-band pulse propagate in a dispersive medium? Explain what happens to a broad-band pulse as it propagates in a dispersive medium.
- In a region of space where there are no sources, derive the wave equation from Maxwell's equation in a non-dispersive medium and determine how the phase velocity depends on the material parameters.
- Define the refractive index of a dielectric and explain how it depends on the material parameters.
- Write down the scalar wave equation and its elementary solutions in terms of plane waves and spherical waves. Explain why they are called plane waves and spherical waves, respectively.
- Explain what is meant by the superposition principle.
- Draw a figure and explain the relation between the $\mathbf{E}$, $\mathbf{H}$, and the propgation direction $\hat{\mathbf{s}}$ for a general electromagnetic plane wave. How are the magnitudes of $\mathbf{E}$ and $\mathbf{H}$ related in the Gaussian system of units?
- Give the relation between the Poynting vector $\mathbf{S}$, the total energy density w, and the propgation direction $\hat{\mathbf{s}}$ for a general electromagnetic plane wave, and explain its physical significance and its dimension.
- Consider a plane, time-harmonic electromagnetic wave propagating in the z direction. Discuss its general state of polarisation and handedness. Under what conditions do we get linear polarisation and circular polarisation?

0.2 Topic 2: Reflection and Transmission of a Plane Electromagnetic Wave at a Plane Interface

- Sketch the derivation by drawing a figure and explain
  - Break up of each of the electric and magnetic incident, reflected and transmitted fields in TE and TM components
  - Boundary conditions
  - Fourier Uniqueness Theorem.
- Explain how the reflection law and Snell's law are obtained and why these laws apply to all kinds of waves. Discuss both the scalar and the vectorial forms of these laws and explain the additional information contained in the vectorial form.
- Define the reflection and transmission coefficients for TE and TM waves. Is there any coupling between these two kinds of plane waves?
- Explain what is meant by the Brewster (polarisation) angle and give the expression for the Brewster angle of incidence θ^{iB}. How are the reflected and the transmitted wave vectors related in this case?
- Explain what is meant by total internal reflection, and give the expression for the critical angle of incidence θ^{ic}. Give a necessary condition to have total internal reflection.
- If the angle of incidence is *smaller* than the critical angle and the incident wave is linearly polarized, what is then the polarisation state of the reflected wave. Explain why.
- If the angle of incidence is *larger* than the critical angle and the incident wave is linearly polarized, what is then the polarisation state of the reflected wave. Explain why.
- Explain how total internal reflection can be used to create circularly polarised light from incident linearly polarised light. Is 1 total reflection sufficient? If not, explain why.

0.3 Topic 3: Planar Boundary Value Problems and Diffraction Problems

- What do we mean by a boundary value problem?
- Given the field in the plane $z = 0$, what is the field in the half-space $z > 0$ in terms of the

  - Angular spectrum of plane waves representation
  - Rayleigh-Sommerfeld's first diffrraction formula u_I.

- Sketch the derivation of the angular-spectrum representation and explain how Rayleigh-Sommerfeld's first diffrraction formula u_I can be obtained from it.

- Explain what is meant by Rayleigh-Sommerfeld's second diffrraction formula u_{II} and Kirchhoff's diffraction formula u_K.

- Given a field that is radiated by a source in the half-space $z < 0$ and that propagates towards the plane $z = 0$. The plane $z = 0$ consist of an opaque screen with an aperture denoted by $\mathcal{A}$. Define the 'Kirchhoff' approximation for u_I, u_{II}, and u_K.

- Which of the formulas u_I, u_{II}, and u_K are consistent in the sense that they give recovery of the boundary values in the plane $z = 0$? What influence does consistency have on the accuracy of the solution?

- Assume that the aperture in the plane $z = 0$ is illuminated by a normally incident plane wave. Explain what we mean by the Fresnel approximation and the Fraunhofer approximation.

- In the Fraunhofer approximation, what kind of matemathical relationship is there between the field in the aperture plane and the field in the observation plane?

0.4 Topic 4a: Diffraction by a Circular Aperture

- Let a plane wave be normally incident upon a circular aperture of radius a in the plane $z = 0$. Discuss the solution u_I for the diffracted field in the

  - Fresnel approximation and the
  - Fraunhofer approximation

  in terms of the dimensionless optical co-ordinates $u = ka^2/z$ and $v = kar/z$, where k is the wave number, z is the axial distance from the aperture to the observation point, and r is the distanse from the axis to the observation point.

- What do we mean by the Airy disc, and what size does it have?

- Explain what we mean by Babinet's principle, and how it can be applied to obtain the obtain the field diffracted by a circular disc of radius a.

- What is Poisson's spot, and what is its physical explanation?

0.5 Topic 4b: Focusing through a Circular Aperture

- Let a converging spherical wave with focus on the axis at $z = z_1$ be diffracted through a circular aperture of radius a in the plane $z = 0$. Discuss the solution u_I for the diffracted field in the
  - Fresnel approximation
  - Fraunhofer approximation

  in terms of the dimensionless optical co-ordinates $u = k(a^2/z_1^2)\tilde{z}$ and $v = kar/z_1$, where k is the wave number, $\tilde{z}$ is the axial distance from the focus to the observation point, and r is the distanse from the axis to the observation point.
- What is the intensity distribuition in the focal plane and along the optical axis?
- Under what conditions do we get a focal shift, implying that the maximum axial intensity is shifted from the focal plane towards the aperture, and what is the physical explanation of it?

0.6 Topic 5: Diffraction by a Half-Plane – Approximate and Exact Solutions

- Let a plane wave be normally incident on the axis of a half-plane extending from $y = 0$ to $y = \infty$ in the plane at $z = 0$. Discuss the approximate 'Kirchhoff' solution obtained using the paraxial approximation and the Fresnel approximation.
- Sketch the intensity distribution in a plane parallel to the half-plane.
- Explain the connection between Huygens' principle, geometrical optics, and edge diffraction.
- Discuss the excact solution, and explain the difference between a 'soft' and a 'hard' boundary condition.
- In the formula for the exact solution, what is the mathematical definition of the detour parameter ξ^i, and what physical significance does it have?
- Under what conditions are the two exact solutions approximately equal to the approximate 'Kirchhoff' solution?