How to get the variational:

We begin with the whole energy:

$$E = T_1 + T_2 + V_1 + V_2 + V_{12}.$$

We know the coulomb-potential and so get for V_i:

$$V_i = -\frac{Ze^2}{r_i}.$$

With the Viral theorem

$$\langle T \rangle = -\frac{1}{2} \langle V \rangle$$

we get for the kinetic energies:

$$T_i = \frac{Ze^2}{2r_i}.$$

We can now then say, that this kinetic energy only depends on the charge that is "seen" from the electrons. Therefore we use the small z instead of the big one and get insofar for the engergy:

$$E = \frac{ze^2}{2r_1} + \frac{ze^2}{2r_2} - \frac{Ze^2}{r_1} - \frac{Ze^2}{r_2} + V_{12}.$$

For a orbit around the core has for the radius the following condition to be fulfilled:

$$r = \frac{n^2}{Z} a_0.$$

This also depends just on the "seen" charge ($Z \to z$) and we have just a look on the ground state ($n = 1$). Taking this radius we get:

$$E = \frac{z^2e^2}{2a_0} + \frac{z^2e^2}{2a_0} - \frac{zZe^2}{a_0} - \frac{zZe^2}{a_0} + V_{12}.$$

Simplified:

$$E = \frac{z^2e^2}{a_0} - \frac{2zZe^2}{a_0} + V_{12}.$$

The repulsion term V_{12} is known for the hydrogen atom:

$$V_{12} = \frac{5}{8}\frac{Ze^2}{a_0}.$$

Here we also have to substitude the big Z with the small one. The reason is, that this term is just influenced by the electrons and these just "feel" the reduced charge.
With this we have all terms:

$$E = z^2\frac{e^2}{a_0} - 2zZ\frac{e^2}{a_0} + \frac{5}{8}z\frac{e^2}{a_0}.$$

By substitution ($E_0 = e^2/a_0$) we get the know formula:

$$E(z, Z) = (z^2 - 2zZ + \frac{5}{8}z) \cdot E_0.$$

If we want to have a stable state the energy have to be extremal:

$$\frac{d}{dz}E(z, Z) = 0$$

$$\Leftrightarrow 2z - 2Z + \frac{5}{8} = 0$$

$$\Leftrightarrow z = Z - \frac{5}{16}.$$