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Errata to Fomin, Lokshtanov, Saurabh, and Zehavi  
“Kernelization” book

The page and statement numbers follow Cambridge's  
final (printed) edition.

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- page xii, 8th line mulivariate -> multivariate

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- page 5, 12th line. Reduction Rule Reduction Rule ->  
Reduction Rule

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- page 6, In the chess example, the knight moves to  
e7 instead of f7

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- page 15p, 5th line. graphs of girth 5 -> graphs of  
girth at least 5

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- page 16, 19th line. Vertex Cover, Feedback Arc Set in Tournaments, Vertex Cover, Feedback Arc Set in Tournaments (FAST) -> Vertex Cover, Feedback Arc Set in Tournaments (FAST)

- page 16, 20th line. graphs of girth 5 -> graphs of girth at least 5  
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- page 22 line 4 of Section 2.4 : "The Dominating Set problem it is known..." => "The Dominating Set problem is known..."  
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- page 23, 10th line. white vertex dominating more than  $k$  black vertices -> white vertex dominating more than  $k - |R|$  black vertices  
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- page 24:  $|B| \leq k^2$  should be  $|B| \leq k(k+1)$ , since one black vertex covers at most  $(k+1)$  black vertices (including itself). The bounds on  $W$  and  $R+W+B$  are also affected.  
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- page 26: In most of reduction rules here, we can use "at least  $|S|$ " instead of "more than  $|S|$ "

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- page 27, 6th line of the proof of Lemma 2.11) a vertex cover of size  $\ell-1$ . -> a vertex cover of size at most  $\ell-1$ .

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- page 27, 11th line of the proof of Lemma 2.11) should belong to  $C$ . -> should belong to  $C'$ .

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- pages 29-30: Problem 2.8: Here  $r$  is a constant that may be hidden by big- $O$ .

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- page 33, Section 3.1 2nd line. "NonBlocker" => "Max Leaf Subtree"

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- page 36 line 4: "u and v become leaves": this is not true, since x or y may coincide with u or v. Here we have to argue that if x coincides with u, then adding xy destroys at most one leaf.

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- page 37, 3rd line of Lemma 3.3. The number of subdivided edges is not at most  $|L|$ , but  $|L|+|B|-1 \leq 2|L|-3$ . This propagates with applications of Lemma 3.3; for instance (3.3) in 44p, 5th line from the bottom of 44p, and 3rd line from the bottom of 46p.

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- page 43: FVS.4 "delete it v" => "delete v"

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- page 43: FVS.5 "none-isolated" => "non-isolated"

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- page 43 line 20 "G'-S contain" => "G'-S contains"

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- page 43. Before reduction FVS.5 we need one more reduction rule. Without it rule FVS.5 is not safe.

Lemma: Let  $v$  be in  $T_1$ , and suppose its neighborhood in  $F$  is a double edge clique  $C$ . Then there exists an optimal FVS  $S$  in  $G$  that avoids  $v$ .

Proof: Let  $S$  be an optimal FVS of  $G$ . If  $v \notin S$  we are done. If  $v$  is in  $S$ , observe that there is at most one vertex  $x$  of  $C$  that is not in  $S$ . Let  $S' = (S - \{v\}) \cup C$ , we have that  $|S'| \leq |S|$ . We claim that  $S'$  is an FVS. Indeed  $v$  has degree at most one in  $G - S'$  and so there are no cycles in  $G - S'$  containing  $v$ . Cycles in  $G - S'$  avoiding  $v$  also avoid  $S$ , contradicting that  $S$  is an FVS.

Corollary: Let  $v$  in  $T_1$  and  $c$  be any vertex in  $C$  adjacent to  $v$  by a double edge. There exists an optimal FVS of  $G$  containing  $c$ .

Pf: take one that avoids  $v$ , it contains  $c$ .

This leads to an extra Reduction Rule FVS.4B: if  $v$  is in  $T_1$  and its neighborhood is a double Edge Clique  $C$ , and  $v$  is adjacent to  $c$  in  $C$  by a double Edge, then remove  $c$  and reduce the parameter  $k$  by 1.

Assuming this rule is applied exhaustively  $v$  is not incident to any double edges and the rule may now be

applied

Proof of Claim 3.12. (Opposite direction.)

Let  $v$  be the vertex of degree 1 in  $T$  whose neighborhood in  $T$  is a double edge clique. Let  $u$  be its neighbor in  $T$ . Let  $w$  be the vertex resulting from the contraction. We want to prove that if  $G/uv$  has a FVS  $S'$  of size at most  $k$  then  $G$  has a FVS of size at most  $k$ .

Let  $S'$  be an FVS of  $G/uv$ .

Case 1:  $S'$  does not contain  $w$ . Then  $S'$  is a vertex set in  $G$ , and we claim that  $S'$  is a FVS in  $G$ . We have that  $(G-S') / uv = (G/uv) - S'$ . So  $(G-S') / uv$  is acyclic. Contracting an edge (when keeping double edges) can not turn a cyclic graph into acyclic, thus  $G-S'$  is acyclic as well.

Case 2:  $S'$  contains  $w$ . Let  $S = (S' - \{w\}) \cup \{u\}$ . We claim that  $S$  is a FVS of  $G$ . Suppose not, and let  $Q$  be a cycle in  $G - S$ . We have that  $G-S = (G-S') \cup v$ . So  $G-S$  is an acyclic graph (namely  $G-S'$ ) plus one vertex  $v$ . So  $Q$  must go through  $v$ . Because of the reduction rule FVS4.a, there are no double edges connecting  $v$  with vertices of  $C$ . Thus the neighbors of  $v$  in  $Q$  are two different vertices. But (since  $u$  is in  $S$ ), both neighbors of  $v$  in  $Q$  are both in  $C$ , contradicting that  $Q$  is an induced cycle (since these two neighbors are connected by a double edge in  $C$  and thus form a cycle in  $G$  and in  $G'$ ). This is a contradiction.

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- page 45 line 11: "To follows our strategy" => "To follow our strategy"

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- page 45: FVS.6 "delete v" => "delete f"

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-page 46 line 18. ... which endpoints -> whose endpoints

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- page 46: Proof of Claim 3.15. "is almost identical to the proof of Claim 3.15"-> Lemma 2.11?

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- page 46, 7th line from the bottom. Since the number of pairs is bounded by  $k(k+1)(2k-1)$ , then the bound  $|T_2|$ ,  $2k(k+1)(2k-1)$  is correct. This propagates to the last two formulas on page 46.

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- page 47: Problem 3.3: This problem is very easy since every graph without isolated vertices has a dominating set of size at most  $n/2$ , so simple kernel with  $2k$  vertices follows. More interesting questions is a  $5/3k$  kernel, from the original paper.

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- page 57, 4th line of Section 4.4. contains a cycle on  $\ell$  vertices.  $\rightarrow$  contains a cycle on at least  $\ell$  vertices.

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- page 58, 2nd line of Problem 4.1. graph  $G$   $\rightarrow$  a graph  $G$

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- page 63, 9th line. from the bottom)  $|B| \geq |A|$ .  $\rightarrow$   $|B| \geq |A|$ .

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- page 63, 2nd line of the proof of Lemma 5.2 "Next, suppose that that..."  $\Rightarrow$  "Next, suppose that..."

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- page 72: in Reduction COC.2: "If  $|B| \geq$ "  $\Rightarrow$  "If  $w(B) \geq$ "

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- page 73, 11th line. size at least  $\ell$ .  $\rightarrow$  size at least  $\ell+1$ .

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- page 73: missing qed in the end of the proof of the claim

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- page 76, 7th line. allowsus  $\rightarrow$  allow us

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- page 77, in (v): "to the the subtree" => "to the subtree"

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- page 79 3rd line: "by more that" => "by more than"

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- page 80 item 3 of Reduction FVS12: "Add edges between  $v$  and vertices in  $S$  such that..." => "Add edges between  $v$  and vertices in  $S$  in such a way that..."

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- page 81 line 14: "such a feedback vertex set the set..." => "such a feedback vertex set..."

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- page 82: For Problem 5.7 a quadratic kernel is easily obtained with simpler techniques. It is better to ask here for a linear kernel.

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- page 91, 10th line of the proof of Theorem 6.5. Conversely, let  $S$  be a minimal vertex cover of size  $k$ . -> Conversely, let  $S$  be a minimal vertex cover of size at most  $k$ .

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- page 97, 9th line from the bottom. not exist a (resp. strictly) reducible pair -> not exist a (resp. strictly) f-reducible pair

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- page 100, 11th line from the bottom.  $f$ -reducible  
-> f-reducible

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- page 112, 2nd line.  $|N_G(A)| \leq |A|$  ->  $|N_{G_{\mathcal{P}}}(A)| \leq |A|$ .  
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- page 112, 3rd line. Define  $Y$  as  $\Delta_{\mathcal{P}} \setminus A$ , instead of  $\Delta_{\mathcal{P}} \setminus \{e \in A\}$ .

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- page 112, 4th line the sets of  $H$  -> the subsets of  $V(H)$

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- page 112, 5th line Consider the following partition of  $H$ , -> Consider the following partition of  $V(H)$ ,  
\$,

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- page 113, 12th line flopping -> flipping

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- page 113, 6th line from the bottom  $H$  is a hypertree ->  $H$  has a hypertree

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- page 115, Reduction IST.1 Otherwise, proceed to Rule IST.3. -> Otherwise, proceed to Rule IST.2.

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- page 117, Theorem 7.13 ISTadmits -> IST admits

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- page 121, 3rd line a an area -> an area

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- page 124, 11th line from the bottom Reduction Rule 8.4 -> Reduction Rule SP.1

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- page 128, 4th line from the bottom: Let us note that since every  $K_{\{d,d\}}$ -free graph is also  $d$ -degenerate... ->Let us note that since every  $d$ -degenerate graph is also  $K_{\{d,d\}}$ -free...

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- page 128, 9th line from the bottom  $\leq$  ->  $\leq$

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- page 129, 3rd line of the proof of Lemma 8.6) Between -> Because

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- page 129, 5th line of the proof of Lemma 8.6  $|F(X)| \leq j$ . ->  $|F(X)| < j$ .

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- page 130, 8th line of the proof of Claim 8.8  $\bigcap_{i=1}^p N(w) \cap I_i$  ->  $\bigcap_{j=1}^p N(w) \cap I_j$

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- page 131, 3rd line  $I = \bigcap_{x \in S} N(u_x)$ , ->  $I = \bigcap_{u_x \in S} N(u_x)$ ,

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- page 131, 17th line Theorem 8.6 -> Lemma 8.6

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- page 136, 2nd line from the bottom in the proof Lemma 9.5 -> in the proof of Lemma 9.5

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- page 139, grey box, Our first observation -> Our first observation

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- page 140, 13th line. "than one unaffected clique" -> "than one unaffected maximal clique."

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- page 141, Figure 9.3. both incident to  $K_1$  and  $K_2$  -> both incident to  $K_1$  or  $K_2$ .

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- page 142, 11th line from the bottom) Observation 9.3 -> Lemma 9.10

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- page 148p, 18th line. "all its children are either leaves or labeled by" -> "each of its children either is a leaf or is labeled by."

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- page 150, Lemma 9.22. Let  $G$  be graph  $\rightarrow$  Let  $G$   
be a graph

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- page 154, 5th line of the proof of Lemma  
9.28. "that do not have both endpoints in  $M$ "  $\rightarrow$   
"that neither of its ends are in  $M$ ."  
(\*235p, 7th line) What is  $\tilde{0}$ ?

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- page 169, 6th line. ... if it forms a spanning  
forest in ...  $\rightarrow$  ... if it forms a forest in ...

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- page 235, 7th line.  $(x, \tilde{0}) \rightarrow (x, k')$ .

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- page 239. The title of section 13.2.1. Should be  
Planar Connected Vertex Cover (not Planar Cluster  
Vertex Deletion)

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- page 159, 4th line. ket  $\rightarrow$  let

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- page 222, inequality in 10th line,  $|T| + k \rightarrow$   
 $|\mathcal{T}| + k$

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- page 234, 8th line. least  $2k$   $\rightarrow$  least  $2k'$

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- page 234, 16th line.  $\geq 2k$   $\rightarrow$   $\geq 2k'$

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- page 237, Theorem 13.1. Then?  $\rightarrow$  Then

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- page 240\_7 Let  $C$  be a \*connected\* vertex cover...

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- page 241^1 Instead of  $|N_2| \leq k$  should be  $|N_2|=0$ .

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- page 260 line 19:  $a \in \chi(\partial T(B))$   
should be  $u \in \chi(\partial T(B))$

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- page 263, 4th line of Lemma 14.18 and 7th line of  
the proof of Lemma 14.18:  $\chi(b') = \chi(b)$ .  $\rightarrow \chi'(b') = \chi(b)$ .

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- page 264, 11th line from the bottom.  $\chi: V(T) \rightarrow V(G)$   
 $\rightarrow \chi: V(T) \rightarrow 2^{V(G)}$

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- page 267, the middle equation. Two  $\setminus$   
between  $\chi$  functions are missed.

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- page 272, 19th line.  $OPT_2 = (OPT \setminus V_{\{b_1\}}) \cup X_2$   
 $\rightarrow OPT_2 = (OPT_1 \setminus V_{\{b_2\}}) \cup X_2$ .

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- page 273, Lemma 14.33.  $k$  is the width of the  
tree decomposition.

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- page 291, Theorem 14.56. Robertson et al. et al.  
 $\rightarrow$  Robertson et al.

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- page 291, Theorem 14.56. For any  $G$  excluding  $H$  as a minor  $\rightarrow$  For any  $G$  excluding a planar  $H$  as a minor

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- page 292 Theorem 14.58. If  $G$  is planar  $\rightarrow$  If  $G$  is a connected planar graph.

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- page 307. line 5 of the proof of Lemma 15.13.  
 $G[\chi(T_{v_i} - M_{i-1})]$  should be  
 $G[\chi(T_{v_i}) \setminus \chi(M_{i-1})]$   
Similar corrections for  
line 7  $G[\chi(T - M_i)] \rightarrow G[\chi(T) \setminus \chi(M_i)]$   
and  
line 15  
 $G[\chi(T_{v_j} - M_{j-1})]$  should be  
 $G[\chi(T_{v_j}) \setminus \chi(M_{j-1})]$

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- pages 310-312 All  $\hat{\phantom{x}}$  symbols over  $T_a$   $\chi$  like in  $(\hat{T}_a, \hat{\chi}_a)$  should be removed.

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- page 313, Theorem 15.22) Dominating Set (vc) on planar graphs  $\rightarrow$  Dominating Set on planar graphs. "

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- page 315, 4th line. Set admits  $2^{o(n)}$  time algorithm -> Set admits no  $2^{o(n)}$  time algorithm  
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- page 328, 2nd line. linear linear -> linear  
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- page 333, Statement of Lemma 16.23

Instead

Moreover, there is an algorithm that, given as input a graph  $G$ , a vertex  $v \in V(G)$  and integers  $p$  and  $q$ , outputs in time

$O(|E(G)| + |V(G)| \cdot \binom{p+q}{p} \cdot (p+q)^{O(1)})$

all pairs  $(P, Q)$  such that  $P$  is a connected vertex set of size  $p+1$  containing  $v$ ,  $Q = N_G(P)$  and  $|Q|=q$ .

it should be

Moreover, there is an algorithm that, given as input a graph  $G$ , and integers  $p$  and  $q$ , outputs in time

$O(|E(G)| + |V(G)| \cdot \binom{p+q}{p} \cdot (p+q)^{O(1)})$

all pairs  $(P, Q)$  such that  $P$  is a connected vertex set of size  $p+1$ ,  $Q = N_G(P)$  and  $|Q|=q$ .

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- page 371, last line: Should be "All but one of its vertices are in  $S$ ."

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- page 437, 14th line from the bottom. is the restriction of  $\Gamma^-\{t,d\}$  -> is the restriction of  $\Gamma_{\{t,d\}}$ .

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- page 438, 4th line, it suffices to given -> it suffices to give

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- page 440, line 13: “no more hold” should be “no longer hold”.

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- page 440, line 14: “setup” should be “set up”.

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- page 441 19th line. instance (I,k) In this chapter -> instance (I,k). In this chapter

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- page 454, Definition 23.17. (fEfficient PSAKS) -> (Efficient PSAKS)

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- page 481, Question of Multicolored Biclique.  $1 \leq i \leq \ell$  ->  $1 \leq i \leq k$