Corrigendum


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It was pointed out to us by Stefan Schreieder and Michael Kemeny that the proof of Proposition 1.2 in [1] contains a gap. More precisely, the gap lies in the four lines after (4), whereas the vanishings on cohomology (4) and (5) remain correct. The latter is used in the proof of Proposition 2.3 in [3].

Firstly, we remark that the gap does not influence any other result in [1]. Proposition 1.2 is only referred to in the proof of Proposition 1.5, to say that \( \dim(W') = g \) in the last but one line, but this is in fact not needed. Indeed, letting \( s \) be the dimension of the general fiber of \( \psi\vert_{V'} \), we have \( g = \dim(V') = s + \dim(W') \) and \( \dim(V) \geq s + \dim(W) \), so that

\[
\dim(V) \geq s + \dim(W) \geq s + \dim(W') + \dim(M_{g,k}^! - \dim(M_g)) = g + (2g + 2k - 5) - (3g - 3) = 2(k - 1),
\]

as stated.

Secondly, although we are at the moment not able to prove Proposition 1.2 in [1], the following weaker statement holds:

**Proposition 0.1.** Let \((S, H) \in \mathcal{K}_p\). For any component \( V \subset V_{(H, \delta)}(S) \) containing irreducible rational nodal curves in its Zariski closure, the restriction \( (\psi_{S,H,\delta})\vert_V \) of the moduli morphism is generically finite. In particular, for a general \((S, H) \in \mathcal{K}_p\), the moduli morphism is generically finite on at least one component of the Severi variety \( V_{(H, \delta)}(S) \).
Proof. For all $0 \leq \delta \leq p$, there is a smooth quasi-projective scheme $\overline{V}_{[H],\delta}(S)$ of pure dimension $p - \delta$ parameterizing pairs $(C, N)$ such that $C \in [H]$ is nodal and irreducible with at least $\delta$ nodes and $N$ is a subset of $\delta$ of its nodes, see, e.g., [4, Thms. 3.8 and 3.11] and [2, §2]. The Severi variety $V_{[H],\delta}(S)$ can be identified with a dense, open subscheme of $\overline{V}_{[H],\delta}(S)$. Furthermore, there is a well-defined extension $\overline{\psi}_{S, H, \delta} : \overline{V}_{[H],\delta}(S) \to \overline{M}_g$ of the moduli morphism mapping a pair $(C, N)$ to the class of the partial normalization of $C$ at $N$.

For integers $0 \leq \delta \leq \delta' \leq p$, there is a quasi-projective subscheme $\overline{V}_{[H],\delta,\delta'}(S) \subseteq \overline{V}_{[H],\delta}(S)$ of pure dimension $p - \delta'$ parameterizing pairs $(C, N)$ such that $C$ has at least $\delta'$ nodes, see again [2, §2]. The regularity of Severi varieties on $K3$ surfaces implies that the nodes of a nodal irreducible curve can be smoothed independently, whence any component of $\overline{V}_{[H],\delta,\delta'+1}(S)$ has codimension one in a component of $\overline{V}_{[H],\delta,\delta'}(S)$, for all $\delta'$. Moreover, the general pair $(C, N)$ in any component of $\overline{V}_{[H],\delta,\delta'}(S)$ is such that $C$ has precisely $\delta'$ nodes.

Let $\overline{V}$ denote the component of $\overline{V}_{[H],\delta}(S)$ containing the curves in $V$. The assumption on $V$ implies that we have a filtration

$$\emptyset \neq \overline{V} \cap \overline{V}_{[H],\delta,p}(S) \subseteq \overline{V} \cap \overline{V}_{[H],\delta,p-1}(S) \subseteq \ldots \subseteq \overline{V} \cap \overline{V}_{[H],\delta,\delta'+1}(S) \subseteq \overline{V}$$

in which every subvariety has codimension one in the subsequent. We may choose for any $i = 0, \ldots, g - 1 = p - \delta - 1$, irreducible components $V_i \subset \overline{V} \cap \overline{V}_{[H],\delta,p-1}(S)$ so as to obtain a filtration

$$\emptyset \neq V_0 \subsetneq V_1 \subsetneq \ldots \subsetneq V_{g-1} \subsetneq \overline{V}$$

by irreducible varieties. Set $W_i := \overline{\psi}_{S, H, \delta}(V_i)$. Then

$$\emptyset \neq W_0 \subsetneq W_1 \subsetneq \ldots \subsetneq W_{g-1} \subsetneq \overline{\psi}_{S, H, \delta}(V).$$

Since $\dim(W_0) = \dim(V_0) = 0$, we have $\dim(W_g) \geq g$, so that equality holds, proving that $\dim \psi_{S, H, \delta}(V) = g$, whence the first assertion.

The last assertion follows by the existence of nodal rational curves in $[H]$ for general $(S, H)$ and regularity. □

References