1 Short introduction

This derivation of ellipsoid coordinates is supplementary to the Path Projection Program (PPP). It discusses, motivates and solves the problem of decimal degrees coordinates, and how to implement it in an $\mathbb{R}^3$ *framework. Note that the problem of the PPP is to properly scale the decimal degree information into meters.*

2 Algorithm details

The spinning Earth warps its spherical nature, and transforms it into the shape of a spheroid (see figure 1).

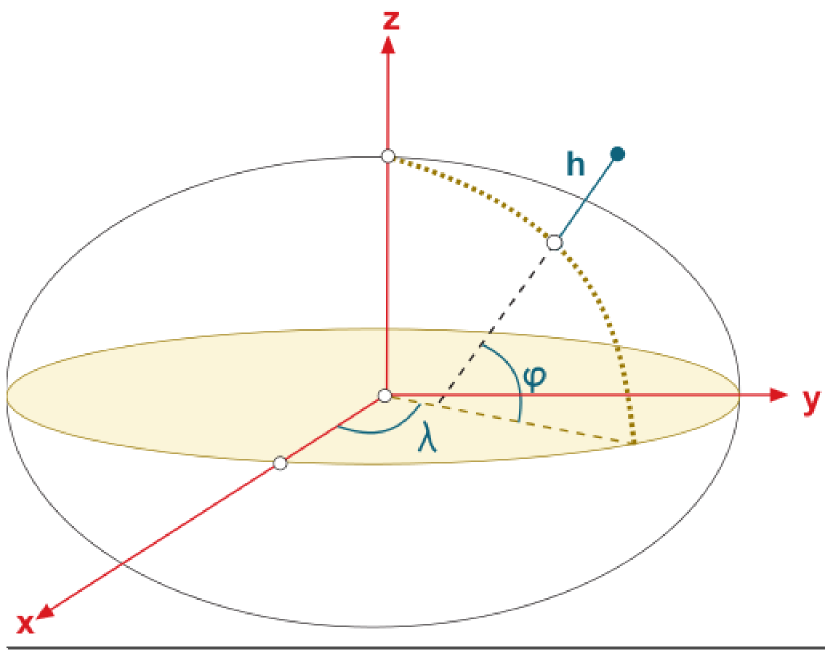

Figure 1: The Cartesian coordinate system in terms of latitude, longitude and height normal to the surface

This means that coordinates in decimal degrees cannot be calculated in a spherical framework, and has to be calculated in a spheroid framework. The way it will be solved is to calculate the Euclidean points, where longitude is denoted by λ, and latitude is denoted by ϕ. Further, the points will be normalized in a way that translates the points from where they are on the globe, to the center (0, 0, 0). Here are formulas relating ellipsoid variables and Cartesian coordinates:

$$x = (N + h)\cos\phi\cos\lambda \tag{1}$$

$$y = (N + h)\cos\phi\sin\lambda \tag{2}$$

$$z = ((1 - e^2)N + h)\sin\phi \tag{3}$$

N is the radius of curvature in the prime vertical:

$$N = \frac{a}{\sqrt{1 - e^2\sin^2\phi}} \tag{4}$$

and the eccentricity e is a constant. For the Earth, the eccentricity is:

$$e^2 = \frac{a^2 - b^2}{a^2} \approx 0.006694378 \tag{5}$$

Recall that a and b denote the semi-major and semi-minor axis, respectively. The image and equations are courtesy of the European Space Agency [1]. Using these equations, it is possible to calculate points in a Cartesian framework. As mentioned previously, the points will be translated such that the zero-point is the point (0, 0, 0). This will be done in a manner of subtracting all points with the values in the zero-coordinate.

To make way for a compass to participate in describing the path, the translated points will have to be transformed such that the x- and y-axes represent the directions east and north, respectively. For this, a matrix transformation was used to tilt the coordinate axes properly. First, let z be stationary:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\lambda & \sin\lambda & 0 \\ -\sin\lambda & \cos\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{6}$$

Further, the y-axis must be rotated:

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} \cos(90-\phi) & 0 & -\sin(90-\phi) \\ 0 & 1 & 0 \\ \sin(90-\phi) & 0 & \cos(90-\phi) \end{bmatrix} \cdot \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \tag{7}$$

The algorithm for producing the normalized points is as follows:

- 1. From the latitude, longitude and elevation points provided by the GPX-data, compute the points' respective Cartesian coordinates. (Be cautious regarding decimals and approximations.)
- 2. Subtract points $(x(t), y(t), z(t))$ from $(x(0), y(0), z(0))$ to translate the point in $t = 0$ to the origin (0, 0, 0).
- 3. Apply an axis rotation transformation to tilt the coordinate system such that the z-axis is fully described by the elevation, provided the zero-translation is done first. *(Steps 2 and 3 are completely interchangable)*

Numbers for calculations are based on WGS84 [2].

References

[1] https://gssc.esa.int/navipedia/index.php/Ellipsoidal_and_Cartesian_Coordinates_Conversion [Retrieved from the Internet 08.06.2020]

[2] https://confluence.qps.nl/qinsy/9.1/en/world-geodetic-system-1984-wgs84-182618391.html:~:text=WGS84%20is%20an%20Earth%2Dcentered,and%20gravity%20and%20geomagnetic%20fields. [Retrieved from the Internet 08.06.2020]